Analytical treatment of quantum systems driven by amplitude-modulated time-periodic force using flow equation approach

Viktor Novičenko (e-mail: viktor.novicenko@tfai.vu.lt)

Institute of Theoretical Physics and Astronomy, Faculty of physics, Vilnius University, Saulėtekio ave. 3, LT-10257 Vilnius, Lithuania

Problem formulation

Quantum system described by a Hamiltonian $h(\omega t+\theta,t)=\sum_{n=-\infty}^{+\infty}\mathrm{e}^{\mathrm{i}n(\omega t+\theta)}h^{(n)}(t)$ which is periodic with respect to the first argument and has additional slow time dependence:

$$\mathrm{i}\hbar\frac{\partial}{\partial t}|\psi_\theta(t)\rangle=h(\omega t+\theta,t)|\psi_\theta(t)\rangle \qquad (*)$$

Expanding $|\psi_\theta(t)\rangle=\sum_n \mathrm{e}^{\mathrm{i}n\theta}|\psi^{(n)}(t)\rangle$ and using extended space approach [1] $\mathscr{L}=\mathscr{T}\otimes\mathscr{H}$, where $\mathrm{e}^{\mathrm{i}n\theta}\equiv|n\rangle\in\mathscr{T}$ is orthonormal basis, we transform Eq (*) in to:

$$\mathrm{i}\hbar\frac{\partial}{\partial t}|\phi(t)\rangle\rangle=\mathcal{K}(t)|\phi(t)\rangle\rangle$$

with $|\phi(t)\rangle\rangle=\mathcal{U}^\dagger|\psi(t)\rangle\rangle=\mathrm{e}^{-\omega t\frac{\mathrm{d}}{\mathrm{d}\theta}}|\psi(t)\rangle\rangle$ and

$$\mathcal{K}(t)=\sum_n|n\rangle n\hbar\omega\langle n|\otimes\mathbf{1}_{\mathscr{H}}+\sum_{n,m}|m\rangle\langle n|\otimes h^{(m-n)}(t) \qquad (\#)$$

The main task is to find block-diagonalizing operator $\mathcal{D}(t)$ such that

$$\mathcal{K}_D(t)=\mathcal{D}^\dagger\mathcal{K}\mathcal{D}-\mathrm{i}\hbar\mathcal{D}^\dagger\frac{\mathrm{d}\mathcal{D}}{\mathrm{d}t}$$

contains non-zero blocks only on a central diagonal [2]:

$$\mathcal{K}_D(t)=\sum_n|n\rangle n\hbar\omega\langle n|\otimes\mathbf{1}_{\mathscr{H}}+\sum_n|n\rangle\langle n|\otimes h_{\mathrm{eff}}(t).$$

Flow towards diagonalization

The main idea of the flow equation approach is to gradually diagonalize some Hamiltonian:

$H(s=0)=$ initial Hamiltonian

↓ run flow equation

$H(s=+\infty)=$ diagonalized Hamiltonian

The flow equation

$$\frac{\mathrm{d}H(s)}{\mathrm{d}s}=[\eta(s),H(s)]$$

with a generator [3] $\eta_{nk}(s)=H_{nk}(s)(H_{nn}(s)-H_{kk}(s))$ is able to diagonalize finite non-degenerate Hamiltonian.

The flow equation to block-diagonalize the extended space Hamiltonian (#)

$$\frac{\partial\mathcal{K}(s,t)}{\partial s}=\mathrm{i}[\mathcal{S}(s,t),\mathcal{K}(s,t)]-\hbar\frac{\partial\mathcal{S}(s,t)}{\partial t}$$

with the generator $\mathrm{i}\mathcal{S}(s,t)=\mathrm{i}\sum_m P_m\otimes S^{(m)}(s,t)$ where a shift operator $P_m=\sum_n|m+n\rangle\langle n|\in\mathscr{T}$ and a m-th Fourier harmonic of the generator

$$\left[S^{(m)}\right]^\dagger=S^{(-m)}\in\mathscr{H}$$

Three posible forms of the generator

(1) For a discrete flow, when s=0,1,2,... , the m-th Fourier harmonic of the extended space Hamiltonian $H^{(m)}(s,t)=\langle m+n|\mathcal{K}(s,t)|n\rangle$ can be expanded as a power series of the inverse frequency $H^{(m)}(s,t)=\sum_{j=0}^{+\infty}H_j^{(m)}(s,t)$, where $H_j^{(m)}\sim\mathcal{O}\left((\hbar\omega)^{-j}\right)$. The main idea is, at each step s, to get rid of the leading order term in the expansion of the non-zero Fourier harmonics of the extended space Hamiltonian. Thus, at the step s, the extended space Hamiltonian

$$\mathcal{K}(s,t)=\hbar\omega N\otimes\mathbf{1}_{\mathscr{H}}+P_0\otimes\sum_{j=0}^{s-1}H_j^{(0)}(s,t)+\mathcal{O}\left((\hbar\omega)^{-s}\right)$$

is diagonal up to the order s-1. It can be realized with the generator of the form

$$\mathrm{i}\mathcal{S}(s,t)=\sum_{m\neq0}\frac{P_m}{m}\otimes\frac{H_s^{(m)}(s,t)}{\hbar\omega}.$$

(2) Continuous flow generator proposed in Ref. [4]

$$\mathrm{i}\mathcal{S}(s,t)=\frac{1}{\hbar\omega}\sum_{m\neq0}mP_m\otimes H^{(m)}(s,t)$$

gives following flow equations:

$$\frac{\mathrm{d}H^{(0)}(s,t)}{\mathrm{d}s}=\frac{2}{\hbar\omega}\sum_{m=1}^{+\infty}m\left[H^{(m)}(s,t),H^{(-m)}(s,t)\right],$$

and

$$\frac{\mathrm{d}H^{(0)}(s,t)}{\mathrm{d}s}=-n^2H^{(n)}(s,t)+\frac{\mathrm{i}}{\omega}h\dot{H}^{(n)}(s,t)+\frac{1}{\hbar\omega}\sum_{m\neq n}(m-n)\left[H^{(m)}(s,t),H^{(n-m)}(s,t)\right].$$

(3) If the initial Hamiltonian does not contain high enough Fourier harmonic, $H^{(m\geq|m_0|)}(s=0,t)=0$, then it would be convenient to have this property on all interval $s\in[0,+\infty)$. In Ref. [5] we propose the generator (generalization of the Toda generator)

$$\mathrm{i}\mathcal{S}(s,t)=\frac{1}{\hbar\omega}\sum_{m\neq0}\mathrm{sgn}(m)P_m\otimes H^{(m)}(s,t)$$

which gives $H^{(m\geq|m_0|)}(s,t)=0$.

References

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