High-frequency analysis of periodically driven quantum system with slowly varying amplitude

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Problem formulation

Quantum system described by the Hamiltonian $H(\omega t + 2\pi, t) = H(\omega t, t)$ which is periodic with respect to the first argument and has additional slow time dependence:

$$i\hbar \frac{\partial}{\partial t} |\phi\rangle = H(\omega t, t) |\phi\rangle \qquad (*)$$

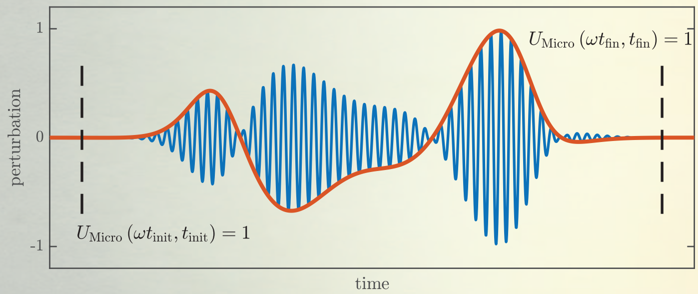
In the limit $\hbar\omega \gg$(any characteristic energy of the system) can be transformed to the Srodinger-type equation with an effective Hamiltonian $H_{\text{eff}}(t)$ does not containing fast oscillations.

Evolution operator

The unitary evolution of equation (*) can be factorized as

$$U(t_{\text{fin}}, t_{\text{init}}) = U_{\text{Micro}}(\omega t_{\text{fin}}, t_{\text{fin}}) U_{\text{eff}}(t_{\text{fin}}, t_{\text{init}}) U^{\dagger}_{\text{Micro}}(\omega t_{\text{init}}, t_{\text{init}})$$

where both: the “Micromotion” operator and the effective evolution operator can be expanded in the powers of $(\hbar\omega)^{-1}$. In most cases the “Micromotion” operator can be ignored, for example if the system is under external perturbation of the form:



$U_{\text{eff}}(t_{\text{fin}}, t_{\text{init}})$ is the effective evolution governed by the Hamiltonian:

$$H_{\text{eff}}(t) = H_{\text{eff}(0)}(t) + H_{\text{eff}(1)}(t) + H_{\text{eff}(2)}(t) + \mathcal{O}(\omega^{-3}),$$

where

$$H_{\text{eff}(0)} = H^{(0)}, \quad H_{\text{eff}(1)} = \frac{1}{\hbar\omega} \sum_{m=1}^{\infty} \frac{1}{m} \left[H^{(m)}, H^{(-m)} \right],$$

$$H_{\text{eff}(2)} = \frac{1}{(\hbar\omega)^2} \sum_{m\neq 0} \left\{ \frac{\left[H^{(-m)}, \left[H^{(0)}, H^{(m)}\right]\right] - i\hbar \left[H^{(-m)}, \dot{H}^{(m)}\right]}{2m^2} + \sum_{n\neq\{0,m\}} \frac{\left[H^{(-m)}, \left[H^{(m-n)}, H^{(n)}\right]\right]}{3mn} \right\}$$

Here the commutators contain the Fourier component of the original Hamiltonian:

$$H(\omega t, t) = \sum_{m=-\infty}^{\infty} H^{(m)}(t) e^{im\omega t}$$

Spin in an oscillating magnetic field

Slowly varying amplitude of the magnetic field:

$$H(\omega t, t) = g_F \underbrace{\mathbf{F}} \cdot \overbrace{\mathbf{B}(t)} \cos(\omega t)$$

Spin operator: $\mathbf{F} = F_1 \mathbf{e}_x + F_2 \mathbf{e}_y + F_3 \mathbf{e}_z$

The non-zero Fourier components are

$$H^{(1)} = H^{(-1)} = \frac{g_F}{2} \mathbf{F} \cdot \mathbf{B}(t)$$

The effective Hamiltonian is given by

$$H_{\text{eff}} = H_{\text{eff}(2)} = \frac{-i\hbar}{(\hbar\omega)^2} \left[H^{(1)}, \dot{H}^{(-1)} \right] = g_F^2 (2\omega)^{-2} \mathbf{F} \cdot \left(\mathbf{B} \times \dot{\mathbf{B}} \right)$$

By definding a non-Abelian geometric vector potential as $\mathcal{A} = g_F^2 (2\omega)^{-2} (\mathbf{F} \times \mathbf{B})$, the effective evolution reads

$$U_{\text{eff}}(t_{\text{fin}}, t_{\text{init}}) = \mathcal{T} \exp \left[-\frac{i}{\hbar} \int_{t_{\text{init}}}^{t_{\text{fin}}} \mathcal{A} \cdot d\mathbf{B}(t) \right] \qquad (**)$$

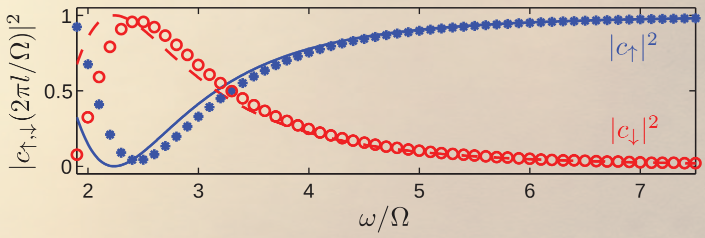
The integral depends only on a shape of the path and does not depend on the velocy. Performing an anticlockwise rotation of the magnetic field with constant amplitude $|\mathbf{B}| = B$ by an angle φ in a plane orthogonal to a unit vector $\mathbf{n} \propto \mathbf{B} \times \dot{\mathbf{B}}$, the evolution operator (**) simplifies to

$$U_{\text{eff}}(\varphi, \mathbf{n}) = \exp \left[-\frac{i}{\hbar} \varphi \frac{g_F^2 B^2}{4\omega^2} \mathbf{F} \cdot \mathbf{n} \right]$$

Comparison of analytical and numerical results for the spin-1/2 particle

$$|\phi(t)\rangle = c_{\uparrow}(t) |\uparrow\rangle + c_{\downarrow}(t) |\downarrow\rangle$$

The magnetic field $\mathbf{B} = \Omega/g_F [\mathbf{e}_z \cos(\Omega t) - \mathbf{e}_y \sin(\Omega t)]$ performs $l = 10$ rotations for the system in an initial state $|\phi\rangle = |\uparrow\rangle$. The analytical results represented by lines while the numerical results depicted by symbols.



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References

[1] V. Novičenko, E. Anisimovas, G. Juzeliūnas: *Phys. Rev. A* **95**, 023615 (2017)