Time-delayed feedback control of periodic orbits with an odd-number of positive unstable Floquet multipliers

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Outline

- Introduction: time-delayed feedback control (TDFC)
- Some scenarios of the stabilization via TDFC
- Odd number limitation
- Comparison of the proportional feedback control and delayed feedback control
- Examples of the successful stabilizations: Lorenz and Chua systems

Time-delayed feedback control

Autonomous system with a unstable periodic orbit (UPO):

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{K}[\mathbf{x}(t-\tau) - \mathbf{x}(t)]$$

The delay time must be equal to the period of UPO.

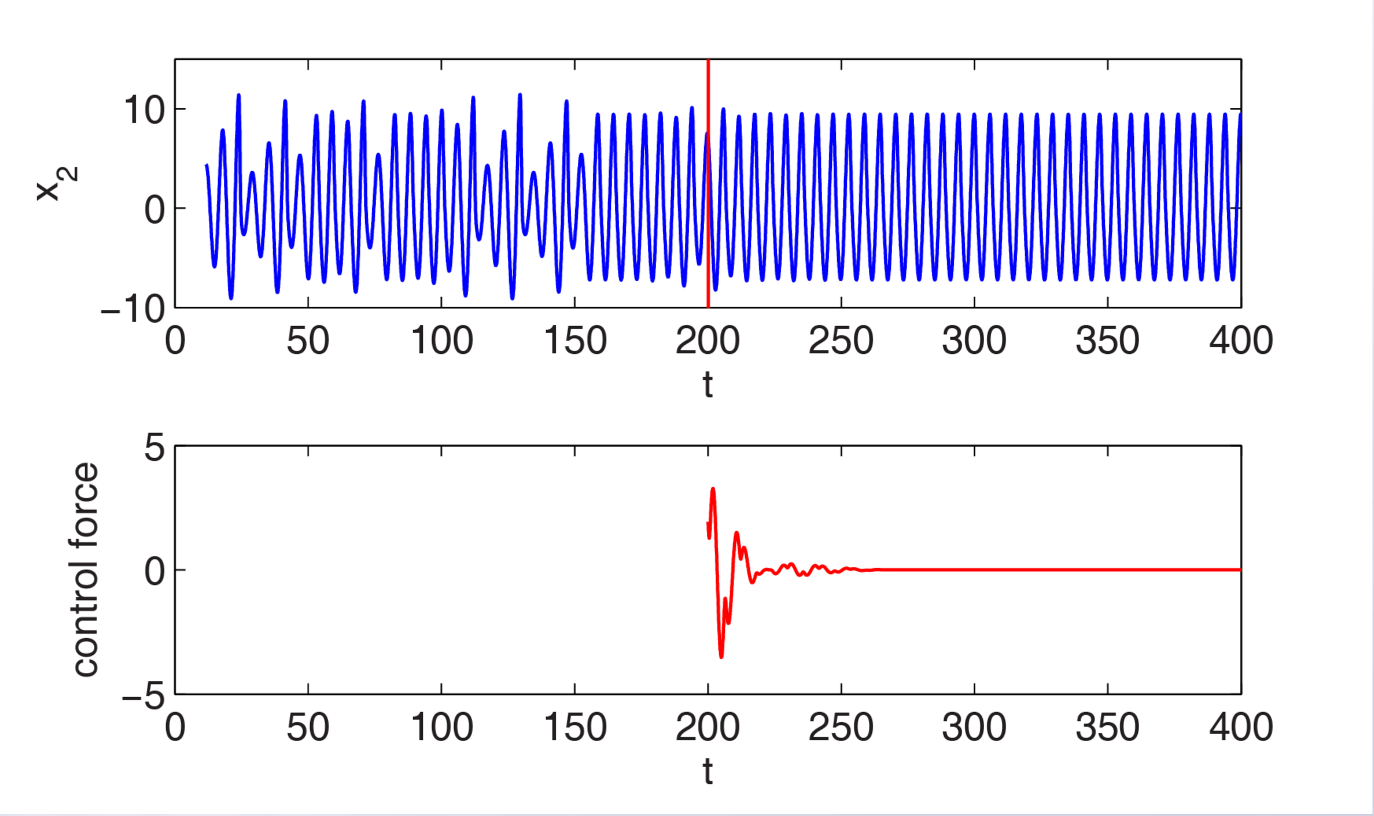
$$\tau = T$$

$$\mathbf{x}_c(t+T) = \mathbf{x}_c(t)$$

noninvasive control force

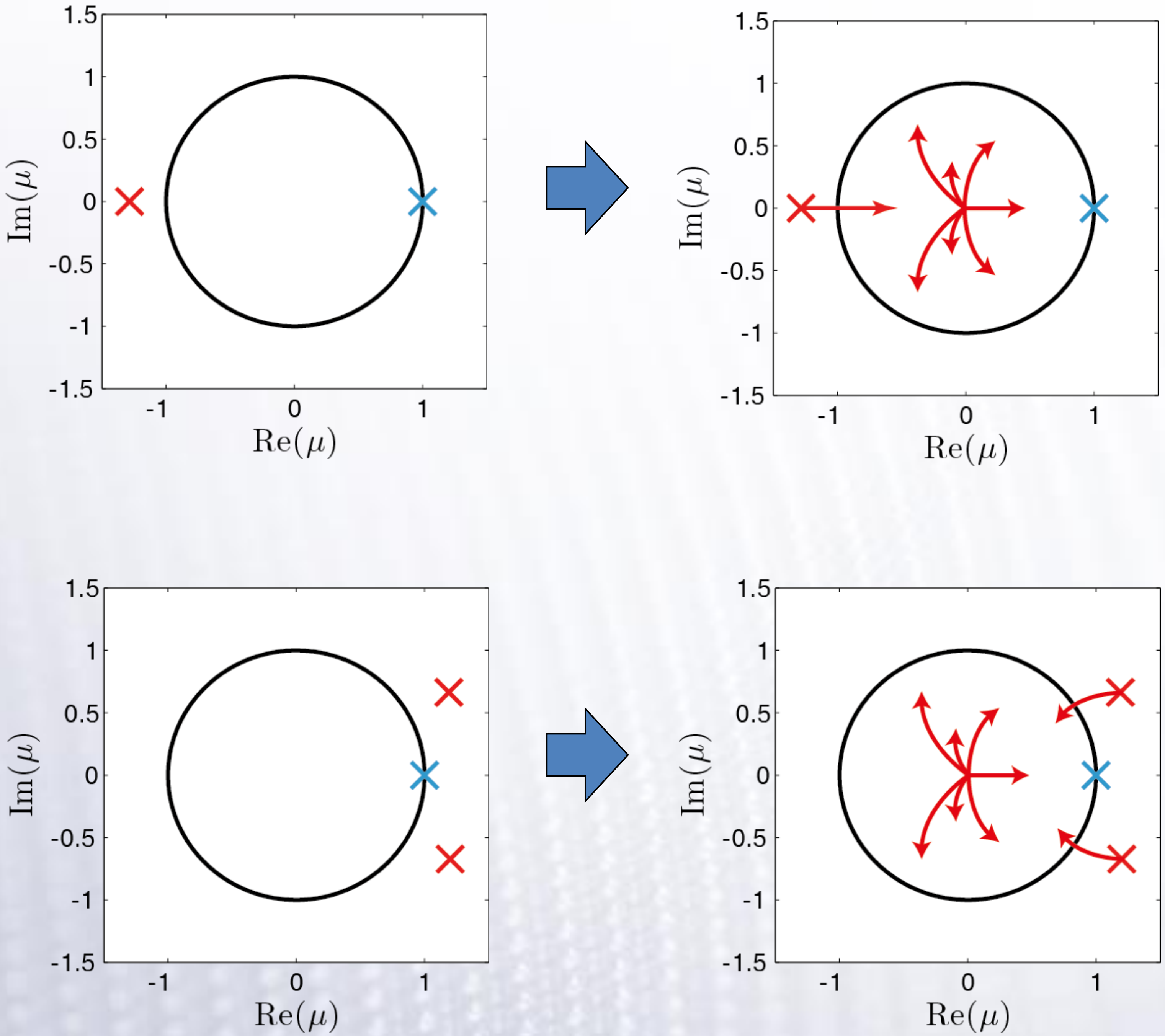
K. Pyragas, Continuous control of chaos by self-controlling feedback, Phys. Lett. A 170 (1992) 421–428

Example of stabilization of period-one UPO in Rossler system:



Some scenarios of the stabilization (I)

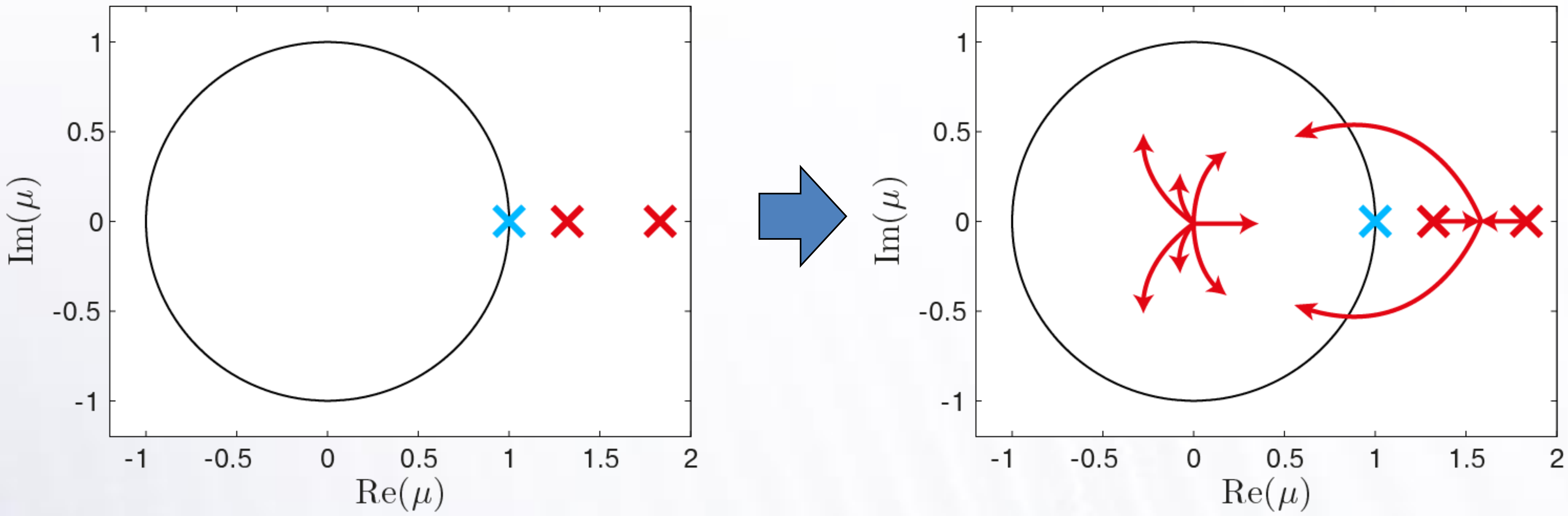
Movement of the Floquet multipliers in the complex plane:



$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{K}[\mathbf{x}(t-\tau) - \mathbf{x}(t)]$$

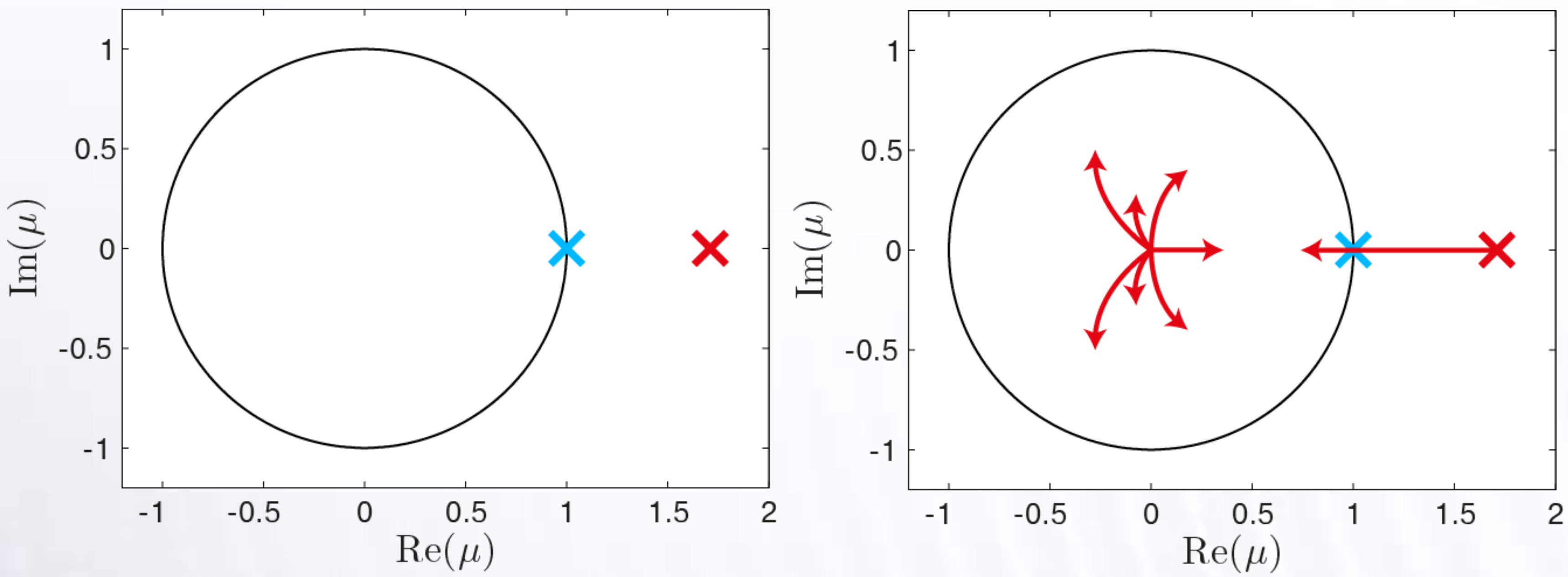
Some scenarios of the stabilization (II)

Movement of the Floquet multipliers in the complex plane:



Some scenarios of the stabilization (III)

Movement of the Floquet multipliers in the complex plane:



m- number of real Floquet multipliers larger than unity in the free system

The orbit is unstable if:

$$(-1)^m \underbrace{\lim_{\tau\to T} \frac{\tau - T}{\tau - \Theta(\mathbf{K},\tau)}}_{\beta} < 0$$

$$\beta = 1 + \sum_{ij} K_{ij} C_{ij}$$

H. Nakajima, Phys. Lett. A **232**, 207 (1997).

B. Fiedler, V. Flunkert, M. Georgi, P. Hovel, and E. Scholl, Phys. Rev. Lett. **98**, 114101 (2007).

E. W. Hooton and A. Amann, Phys. Rev. Lett. **109**, 154101 (2012).

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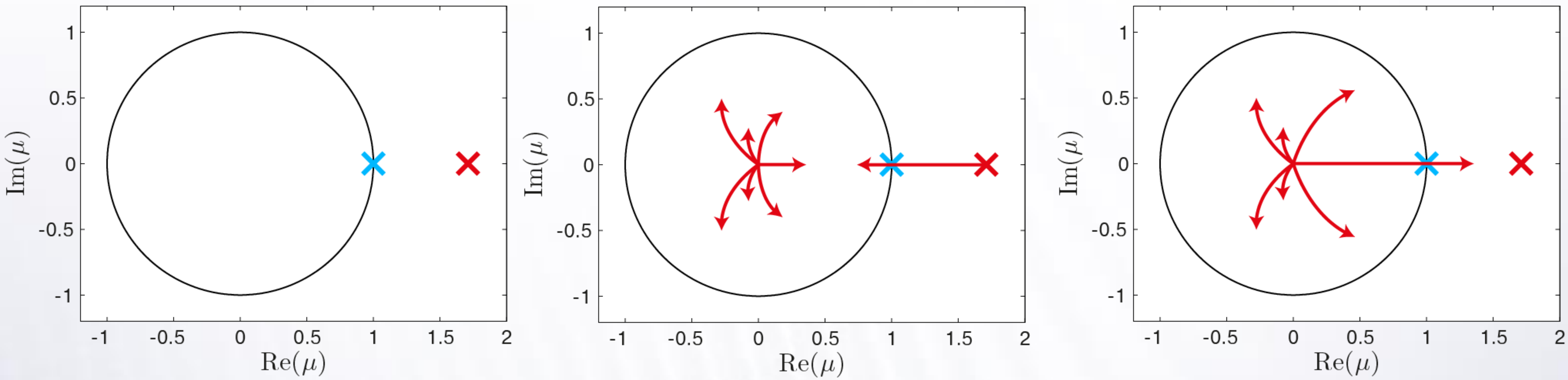

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Comparison of the proportional and the delay feedback controls in the Lorenz system

Proportional feedback control (PFC):

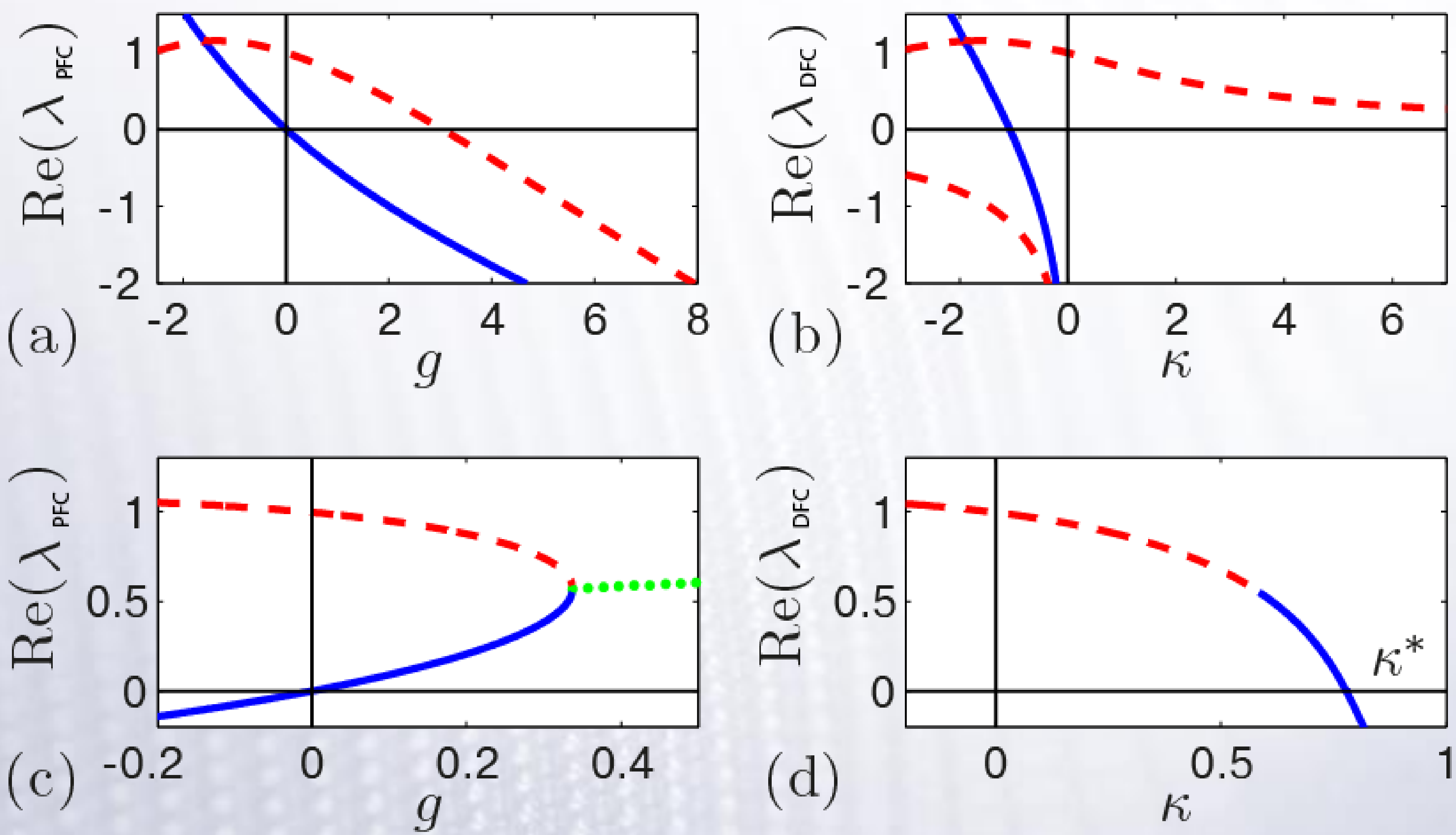
$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + g\tilde{\mathbf{K}}[\mathbf{x}_c(t) - \mathbf{x}(t)]$$

Delay feedback control (DFC):

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \kappa\tilde{\mathbf{K}}[\mathbf{x}(t-\tau) - \mathbf{x}(t)]$$

$$\lambda_{DFC} = \lambda_{PFC}(g)$$

$$\kappa = \frac{g}{1 - \exp(-\lambda_{PFC}(g)T)}$$



$$\tilde{\mathbf{K}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

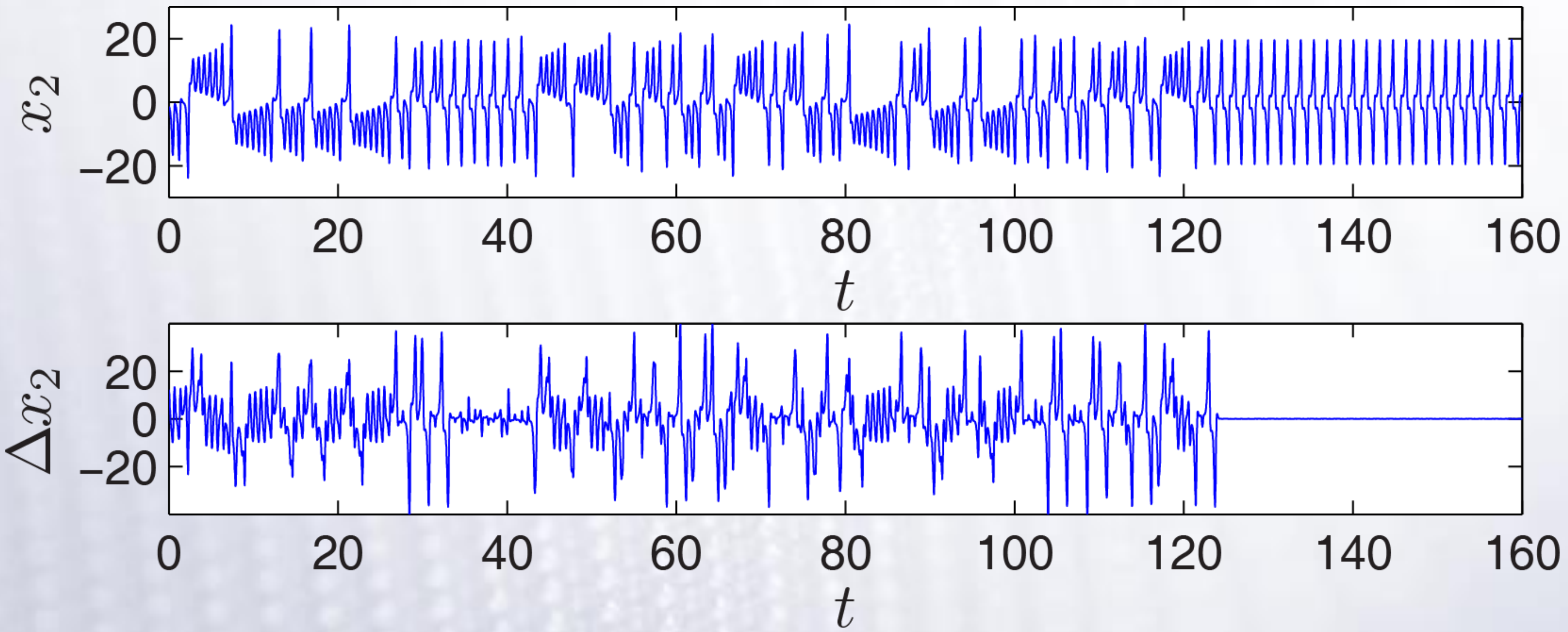
$$\tilde{\mathbf{K}} = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0.5 \\ 0 & 0 & 0 \end{bmatrix}$$

Successful stabilization: Lorenz system

$$\dot{x}_1 = 10[x_2 - x_1]$$
$$\dot{x}_2 = x_1[28 - x_3] - x_2$$
$$\dot{x}_3 = x_1 x_2 - 8/3\, x_3$$

$$\tilde{\mathbf{K}} = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0.5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\kappa = 0.865$$



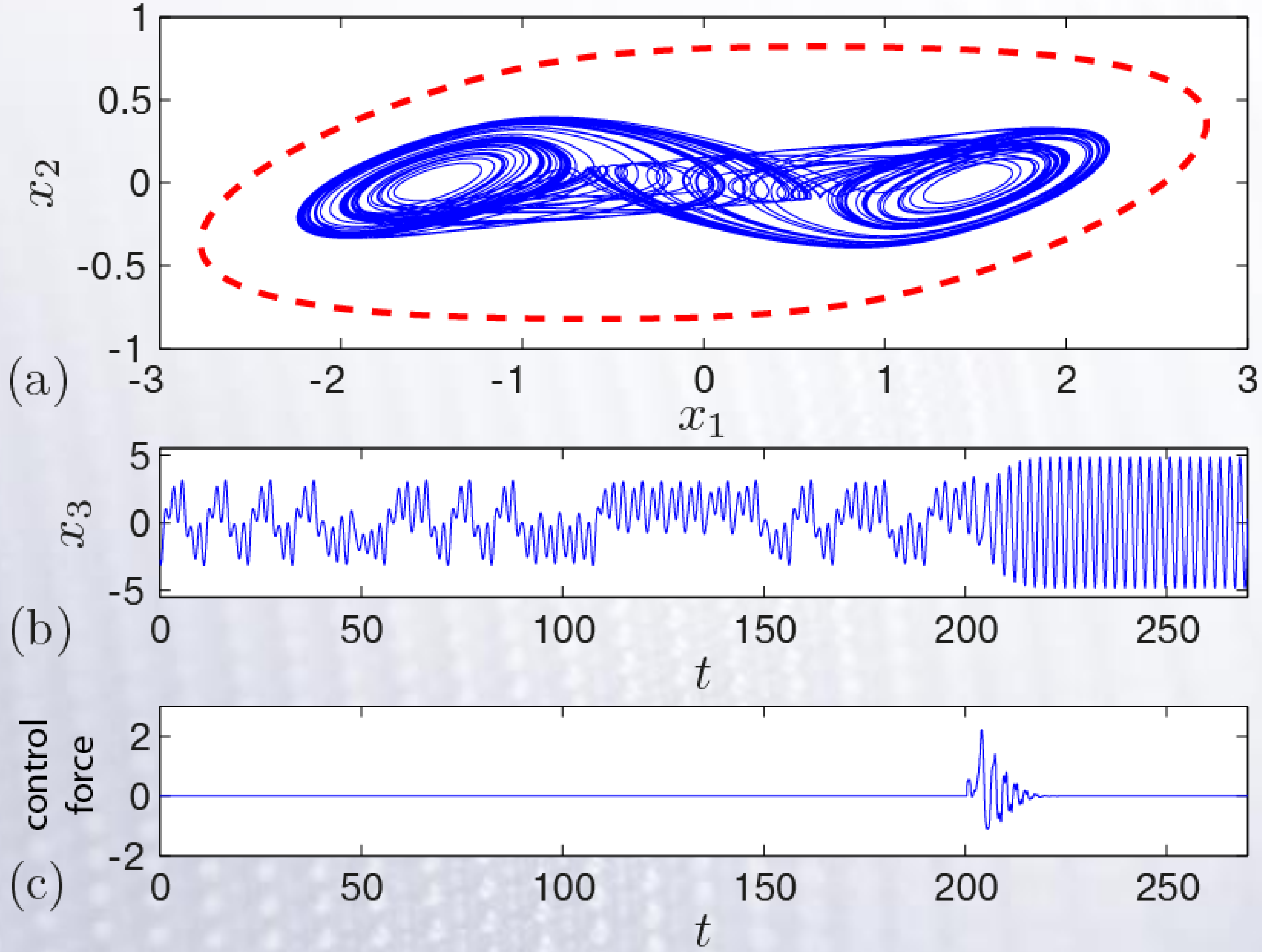
$$\Delta x_2 = x_2(t) - x_2(t - \tau)$$

Successful stabilization: Chua system

$$\dot{x}_1 = 9\left(x_2 - \phi(x)\right)$$
$$\dot{x}_2 = x_1 - x_2 + x_3$$
$$\dot{x}_2 = -\frac{100}{7}x_2$$

Nonlinear function:

$$\phi(x) = \frac{2}{7}x_1 - \frac{3}{14}\left(|x_1+1| - |x_1-1|\right)$$



$$\tilde{K} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0.3 \end{bmatrix}$$

$$\kappa = 1.2$$

Acknowledgement

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Thank you for your attention