Floquet analysis of a quantum system with modulated periodic driving

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Introduction about Lithuania

Lithuania on a world map

Lithuania has borders with:

- Latvia
- Belarus
- Poland
- Russia

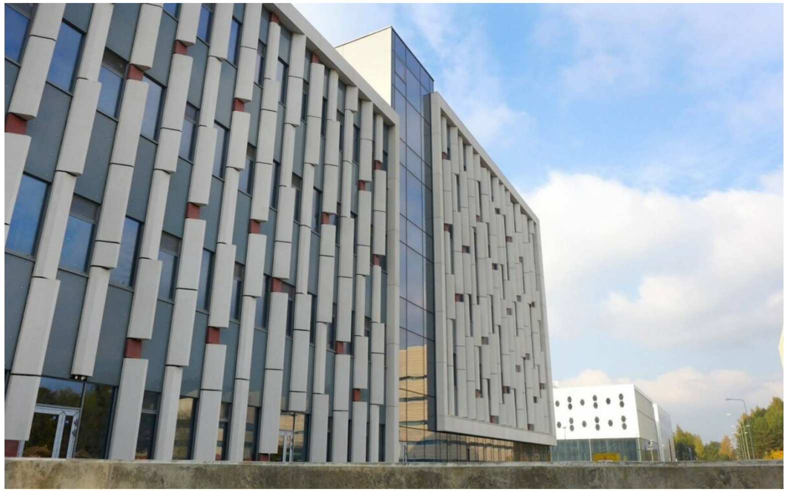
From 2004 Lithuania is a member of European union. From 2015 euro became the national currency.

Capital of Lithuania - Vilnius, population about 570 000

Vilnius university

Vilnius university was founded in 1579

Center for physical science and technology



Main message

$$i\hbar \frac{\partial}{\partial t}\left|\phi\left(t\right)\right\rangle = H\left(\omega t, t\right)\left|\phi\left(t\right)\right\rangle$$

periodic dependence with respect to the first argument $H\left(\omega t + 2\pi, t\right) = H\left(\omega t, t\right)$

$\hbar\omega \gg$ any other characteristic energies of the system, or in other words

$$H\left(\omega t, t\right) = \sum_{n=-\infty}^{\infty} H^{(n)}\left(t\right) e^{in\omega t}$$

matrix elements $\left|H^{(n)}_{\alpha\beta}\right| \ll \hbar\omega$

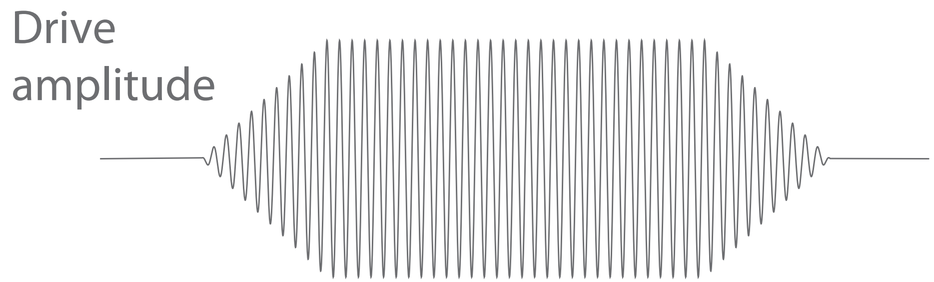
derivative of matrix elements $\left|\dot{H}^{(n)}_{\alpha\beta}\right| \ll \left|H^{(n)}_{\alpha\beta}\right|\omega$

$$i\hbar \frac{\partial}{\partial t}\left|\phi\left(t\right)\right\rangle = H_{\text{eff}}\left(t\right)\left|\phi\left(t\right)\right\rangle$$

V. Novičenko, E. Anisimovas, G. Juzeliūnas: *Phys. Rev. A* **95**, 023615 (2017)

Motivation

Shaken optical lattice



R. Desbuquois, M. Messer, F. Görg, K. Sandholzer, G. Jotzu, T. Esslinger: *Phys. Rev. A* **96**, 053602 (2017)

Extension of the space

Let us study whole family of the solutions:

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\left|\phi_\theta(t)\right\rangle = H(\omega t+\theta, t)\left|\phi_\theta(t)\right\rangle \qquad \theta\in[0,2\pi]$$

with initial conditions

$$\left|\phi_{\theta+2\pi}(t_{\mathrm{in}})\right\rangle = \left|\phi_\theta(t_{\mathrm{in}})\right\rangle$$

Hamiltonian $H(\omega t+\theta, t)$ acts on a Hilbert space $\mathcal{H}$

Introduce the space $\mathcal{T}$ of θ-periodic functions

Construct the space $\mathcal{L}=\mathcal{H}\otimes\mathcal{T}$

Apply unitary transformation $U=\exp\left[\omega t\frac{\partial}{\partial\theta}\right]$

$$K = U^\dagger H U - \mathrm{i}\hbar U^\dagger\frac{\mathrm{d}U}{\mathrm{d}t} = -\mathrm{i}\hbar\omega\frac{\partial}{\partial\theta} + H(\theta, t)$$

$$K = \sum_{n=-\infty}^{\infty}\left|\overline{n}\right\rangle n\hbar\omega\mathbf{1}\left\langle\overline{n}\right| + \sum_{n,m=-\infty}^{\infty}\left|\overline{m}\right\rangle H^{(m-n)}(t)\left\langle\overline{n}\right|$$

Orthonormal basis of the space $\mathcal{T}$

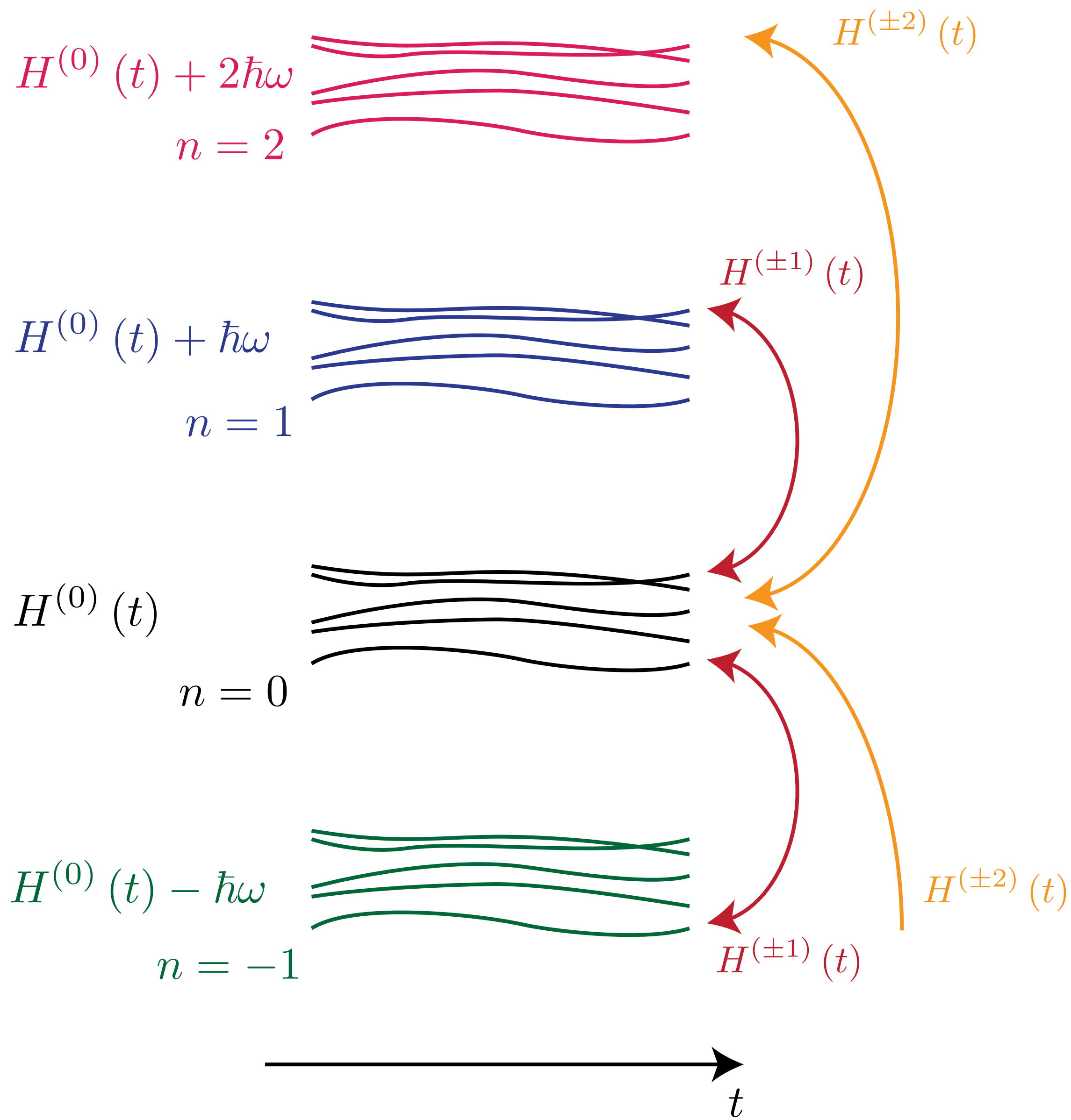
$$\frac{e^{\mathrm{i}n\theta}}{\sqrt{2\pi}}\leftrightarrow\left|\overline{n}\right\rangle$$

where the Fourier expansion of the Hamiltonian $H(\theta,t)=\sum_{n=-\infty}^{\infty}H^{(n)}(t)\,e^{\mathrm{i}n\theta}$

Matrix representation of the "Kamiltonian" operator

$$K(t) = \begin{pmatrix} & \vdots & \vdots & \vdots & \\ \cdots & H^{(0)}(t) - \hbar\omega\mathbf{1} & H^{(-1)}(t) & H^{(-2)}(t) & \cdots \\ \cdots & H^{(1)}(t) & H^{(0)}(t) & H^{(-1)}(t) & \cdots \\ \cdots & H^{(2)}(t) & H^{(1)}(t) & H^{(0)}(t) + \hbar\omega\mathbf{1} & \cdots \\ & \vdots & \vdots & \vdots & \end{pmatrix}$$

Floquet band structure of the "Kamiltonian" operator



Block diagonalization of the "Kamiltonian"

$$K_D = D^\dagger(t)\,K(t)\,D(t) - \mathrm{i}\hbar D^\dagger(t)\,\dot{D}(t) = \sum_{n=-\infty}^{\infty} |\overline{n}\rangle \left(H_{\text{eff}}(t) + n\hbar\omega\mathbf{1}\right)\langle\overline{n}|$$

$$K_D = \begin{array}{ccccc} & \vdots & \vdots & \vdots & \\ \cdots & H_{\text{eff}}(t) - \hbar\omega\mathbf{1} & \mathbf{0} & \mathbf{0} & \cdots \\ \cdots & \mathbf{0} & H_{\text{eff}}(t) & \mathbf{0} & \cdots \\ \cdots & \mathbf{0} & \mathbf{0} & H_{\text{eff}}(t) + \hbar\omega\mathbf{1} & \cdots \\ & \vdots & \vdots & \vdots & \end{array}$$

High-frequency expansion

$$D(t) = \sum_{n=-\infty}^{\infty} |\overline{n}\rangle \mathbf{1} \langle\overline{n}| + D_{(1)}(t) + D_{(2)}(t) + \mathcal{O}\left(\omega^{-3}\right)$$

$$H_{\text{eff}}(t) = H_{\text{eff}(0)}(t) + H_{\text{eff}(1)}(t) + H_{\text{eff}(2)}(t) + \mathcal{O}\left(\omega^{-3}\right)$$

$$D^{\dagger}(t) K(t) D(t) - \mathrm{i}\hbar D^{\dagger}(t) \dot{D}(t) = \sum_{n=-\infty}^{\infty} |\overline{n}\rangle \left(H_{\text{eff}}(t) + n\hbar\omega\mathbf{1}\right) \langle\overline{n}|$$

$$H_{\text{eff}(0)} = H^{(0)}$$

$$H_{\text{eff}(1)} = \frac{1}{\hbar\omega} \sum_{m=1}^{\infty} \frac{1}{m} \left[H^{(m)}, H^{(-m)}\right]$$

$$H_{\text{eff}(2)} = \frac{1}{(\hbar\omega)^2} \sum_{m\neq 0} \left\{ \frac{\left[H^{(-m)}, \left[H^{(0)}, H^{(m)}\right]\right] - \mathrm{i}\hbar \left[H^{(-m)}, \dot{H}^{(m)}\right]}{2m^2} + \sum_{n\neq\{0,m\}} \frac{\left[H^{(-m)}, \left[H^{(m-n)}, H^{(n)}\right]\right]}{3mn} \right\}$$

$$\blacksquare = 0 \qquad \blacksquare \neq 0$$

Our original problem:

$$\mathrm{i}\hbar \frac{\partial}{\partial t} |\phi_\theta(t)\rangle = H(\omega t + \theta, t) |\phi_\theta(t)\rangle$$

$$|\phi_\theta(t_{\text{fin}})\rangle = U_{\text{Micro}}(\omega t_{\text{fin}} + \theta, t_{\text{fin}}) U_{\text{eff}}(t_{\text{fin}}, t_{\text{in}}) U^{\dagger}_{\text{Micro}}(\omega t_{\text{in}} + \theta, t_{\text{in}}) |\phi_\theta(t_{\text{in}})\rangle$$

$$\mathrm{i}\hbar \frac{\partial}{\partial t} |\chi(t)\rangle = H_{\text{eff}}(t) |\chi(t)\rangle$$

Spin in an oscillating magnetic field

The system Hamiltonian

$$H\left(\omega t, t\right) = g_F \mathbf{F} \cdot \mathbf{B}\left(t\right) \cos\left(\omega t\right)$$

The non-zero Fourier components

$$H^{(1)}\left(t\right) = H^{(-1)}\left(t\right) = \frac{g_F}{2} \mathbf{F} \cdot \mathbf{B}\left(t\right)$$

The effective Hamiltonian is non-zero only due to “slow” time derivative

$$H_{\mathrm{eff}}\left(t\right) = H_{\mathrm{eff}(2)}\left(t\right) = -\frac{\mathrm{i}\hbar}{\left(\hbar\omega\right)^2}\left[H^{(1)}, \dot{H}^{(1)}\right] = \mathbf{A} \cdot \dot{\mathbf{B}}$$

where $\mathbf{A} = \frac{g_F^2}{2\omega^2} \mathbf{F} \times \mathbf{B}$ is a geometric matrix valued non-Abelian vector potential

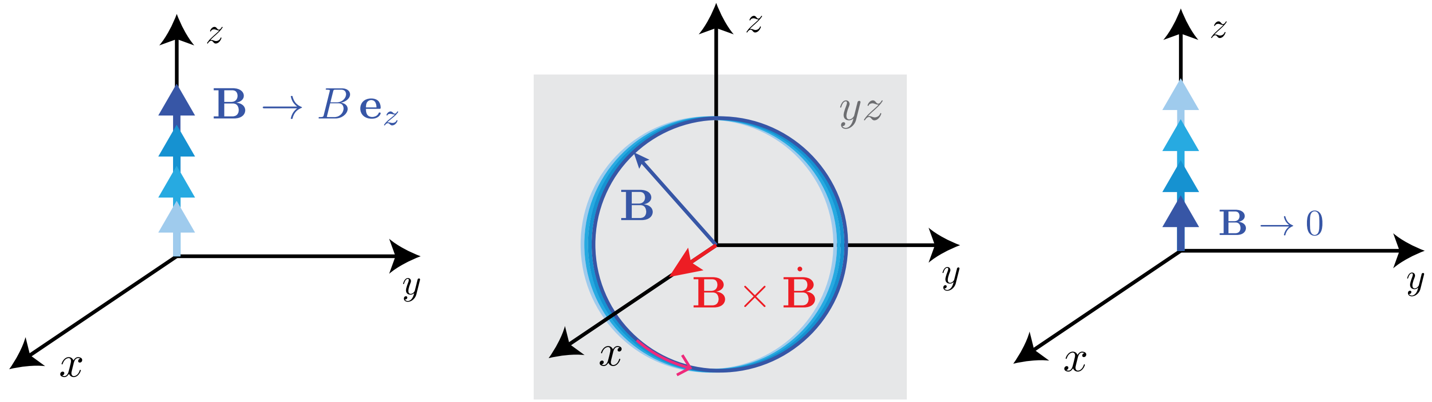
The effective evolution

$$U_{\mathrm{eff}}\left(t_{\mathrm{fin}}, t_{\mathrm{in}}\right) = \mathcal{P} \exp\left[-\frac{\mathrm{i}}{\hbar} \int_{\mathbf{B}(t_{\mathrm{in}})}^{\mathbf{B}(t_{\mathrm{fin}})} \mathbf{A} \cdot \mathrm{d}\mathbf{B}\left(t\right)\right]$$

If $\left|\mathbf{B}\left(t\right)\right| = B = \mathrm{const}$ and performs rotationin a plane by an ange φ

$$U_{\mathrm{eff}}\left(\mathbf{n}, \varphi\right) = \exp\left[-\frac{\mathrm{i}}{\hbar}\gamma\left(\varphi\right) \mathbf{F} \cdot \mathbf{n}\right]$$, where $\gamma\left(\varphi\right) = \varphi \frac{g_F^2 B^2}{4\omega^2}$ and $\mathbf{n} = \frac{\mathbf{B} \times \dot{\mathbf{B}}}{\left|\mathbf{B} \times \dot{\mathbf{B}}\right|}$

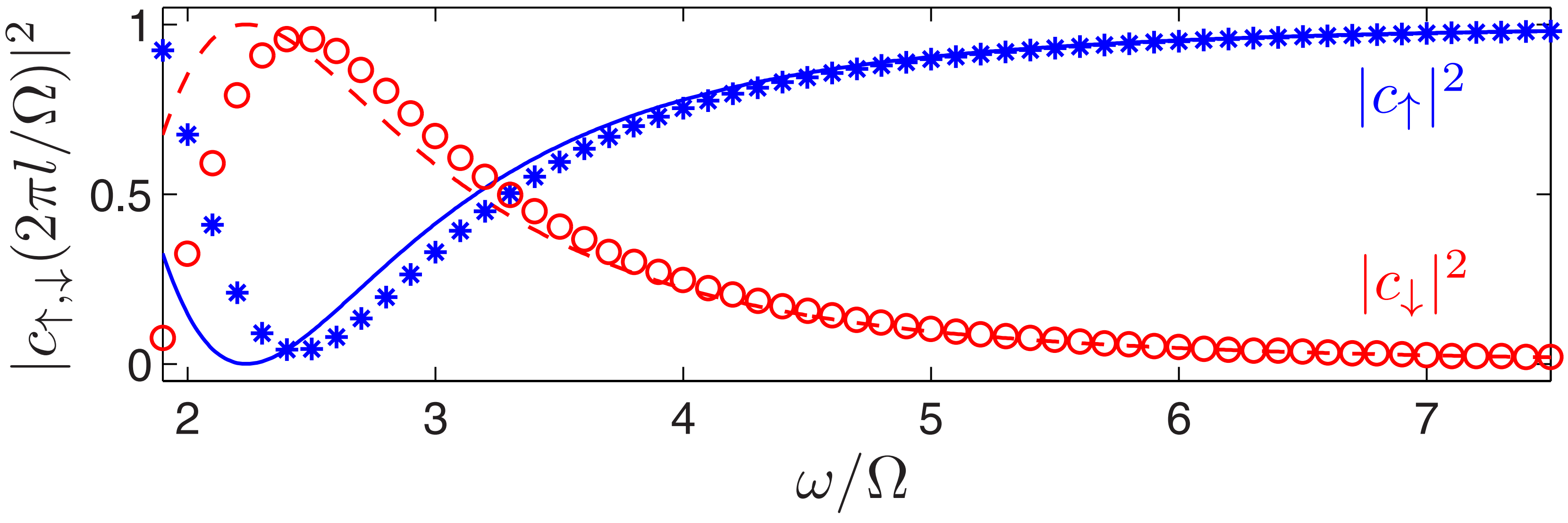
Numerical demonstration of a spin-1/2 particle



$$\mathbf{B}\,(t) = B\left[\mathbf{e}_z \cos\left(\Omega t\right) - \mathbf{e}_y \sin\left(\Omega t\right)\right]$$

The amplitude of the magnetic field performs $l = 10$ rotations

$|\phi\,(t)\rangle = c_{\uparrow}\,(t)\,|{\uparrow}\rangle + c_{\downarrow}\,(t)\,|{\downarrow}\rangle$ initial conditions $c_{\uparrow}\,(t_{\text{in}}) = 1,\quad c_{\downarrow}\,(t_{\text{in}}) = 0$



Spin in a strong oscillating magnetic field

Same Hamiltonian

$H(\omega t, t) = g_F \mathbf{F} \cdot \mathbf{B}(t) \cos(\omega t)$ but now $g_F \mathbf{F} \cdot \mathbf{B}(t) \sim \hbar\omega$ while $\frac{|\dot{\mathbf{B}}|}{|\mathbf{B}|} \ll \omega$

Apply unitary transformation $R(\omega t, t) = \exp\left[-\frac{\mathrm{i}}{\hbar\omega} \sin(\omega t)\, g_F \mathbf{F} \cdot \mathbf{B}(t)\right]$

$$W(\omega t, t) = R^\dagger H R - \mathrm{i}\hbar R^\dagger \frac{\mathrm{d}R}{\mathrm{d}t}$$

The zero-order effective Hamiltonian $W_{\mathrm{eff}(0)}(t) = \mathbf{A} \cdot \dot{\mathbf{B}}$ with vector potential

$$\mathbf{A} = \frac{[1 - \mathcal{J}_0(g_F B/\omega)]}{B^2} \mathbf{F} \times \mathbf{B}$$

If $|\mathbf{B}(t)| = B = \mathrm{const}$ and performs rotationin a plane by an ange φ

$U_{\mathrm{eff}}(\mathbf{n}, \varphi) = \exp\left[-\frac{\mathrm{i}}{\hbar}\gamma(\varphi)\, \mathbf{F} \cdot \mathbf{n}\right]$, where $\gamma(\varphi) = \varphi\,[1 - \mathcal{J}_0(g_F B/\omega)]$ and $\mathbf{n} = \frac{\mathbf{B} \times \dot{\mathbf{B}}}{|\mathbf{B} \times \dot{\mathbf{B}}|}$

V. Novičenko, G. Juzeliūnas: *Phys. Rev. A* **100**, 012127 (2019)

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