Adaptive delayed feedback control to stabilize in-phase synchronization in complex oscillator networks

Viktor Novičenko

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Vilnius University

Motivation

Synchronous behavior can be desirable or harmful.

- Power grids
- Parkinson’s disease, essential tremor
- Pedestrians on a bridge
- Cardiac pacemaker cells
- Internal circadian clock

The ability to control synchrony in oscillatory network covers a wide range of real-world applications.

Complex oscillator network - the phase reduction approach

Weakly coupled near-identical limit cycle oscillators **without control**:

$$\dot{\mathbf{x}}_i = \mathbf{f}_i\left(\mathbf{x}_i\right) + \varepsilon \sum_{j=1}^{N} a_{ij} \mathbf{G}\left(\mathbf{x}_j, \mathbf{x}_i\right)$$

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effective frequency

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the function $\alpha(KC) = (1 + KC)^{-1}$ and the constant $C = \int_0^T \left\{ \mathbf{z}^T(s) \cdot D_2 \mathbf{f}(\boldsymbol{\xi}(s), 0) \right\} \left\{ [\nabla g(\boldsymbol{\xi}(s))]^T \cdot \dot{\boldsymbol{\xi}}(s) \right\} \mathrm{d}s$

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Control of synchronization by delayed feedback force

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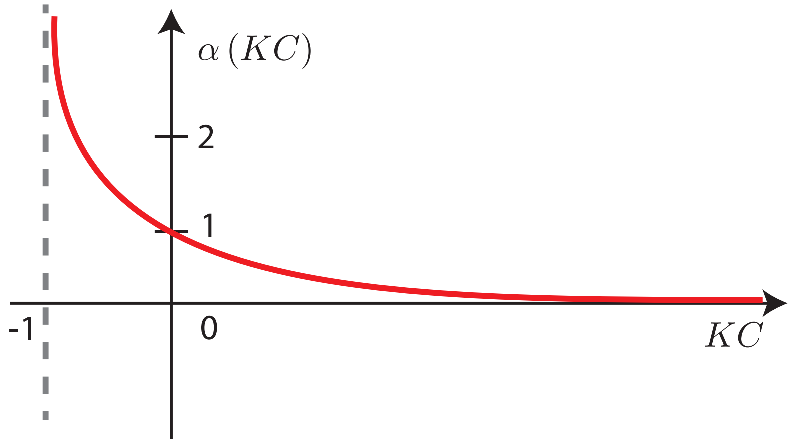

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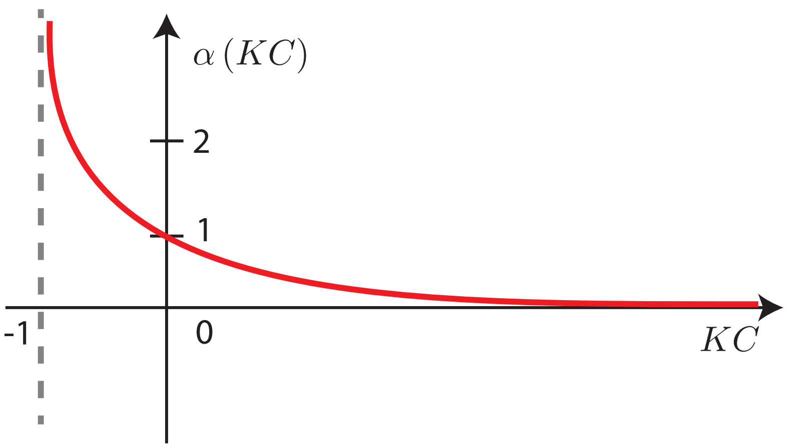

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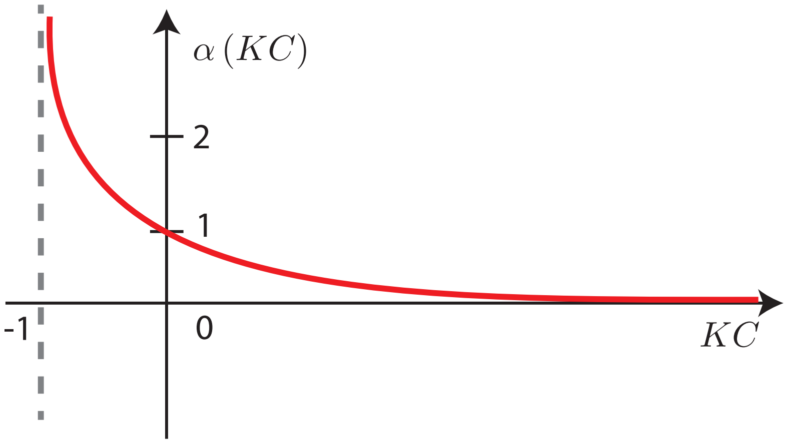

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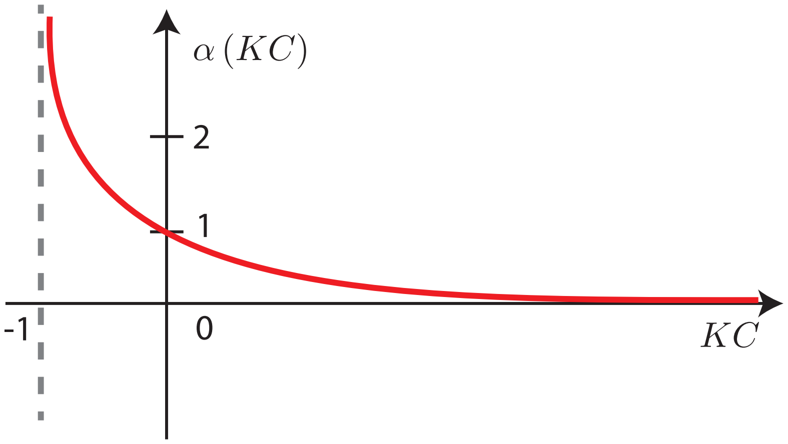

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in-phase synchronization $\psi_1 = \psi_2 = \ldots = \psi_N$ is a stable solution of Eq. (1)

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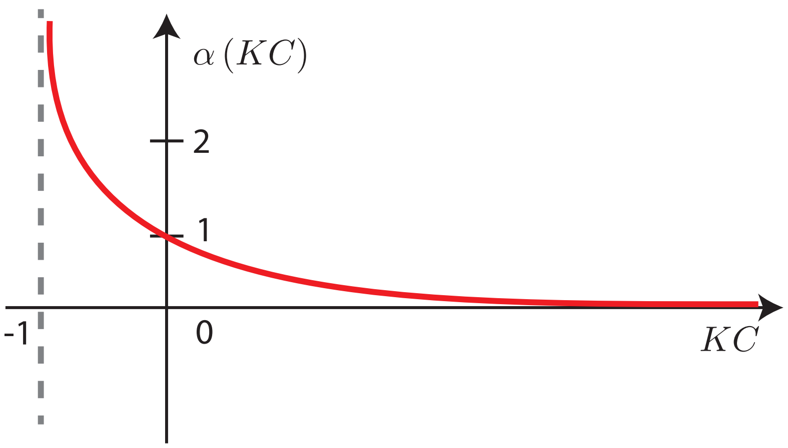

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Our goal is to derive an algorithm for authomatic adjusment of the time delays to acheve in-phase synchronization

Adaptive delayed feedback control to stabilize in-phase synchronization

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potential for the in-phase synchronization

$$V = \frac{1}{2} \sum_{i,j=1}^{N} a_{ij} \left[s_j(t) - s_i(t)\right]^2$$

V. Novičenko and I. Ratas: **In-phase synchronization in complex oscillator networks by adaptive delayed feedback control**, *Phys. Rev. E* **98**, 042302 (2018)

Adaptive delayed feedback control to stabilize in-phase synchronization

V. Pyragas and K. Pyragas: **Adaptive modification of the delayed feedback control algorithm with a continuously varying time delay**, *Phys. Lett. A* **375**, 3866 (2011)

Main idea:

- construct potential $V \geq 0$ which is equal to zero at desirable state and positive at other states
- use gradient descent method for the time delay $\dot{\tau} = -\frac{\partial V}{\partial \tau}$

$$\dot{\mathbf{x}}_i = \mathbf{f}_i\left(\mathbf{x}_i, u_i(t)\right) + \varepsilon \sum_{j=1}^{N} a_{ij} \mathbf{G}\left(\mathbf{x}_j, \mathbf{x}_i\right)$$

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$$\dot{\psi}_i = \omega_i^{\text{eff}} + \varepsilon^{\text{eff}} \sum_{j=1}^{N} a_{ij} h\left(\psi_j - \psi_i\right)$$

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finally the derivative $\frac{\partial \psi_i}{\partial \tau_k} \sim \left(\mathbf{L}^{\dagger}\right)_{ik}$

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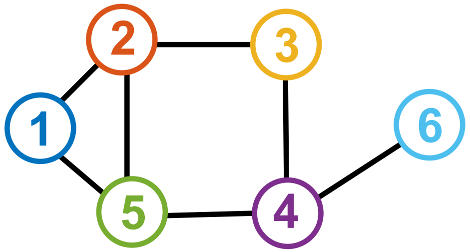
Network of 6 Stuart-Landau oscillators

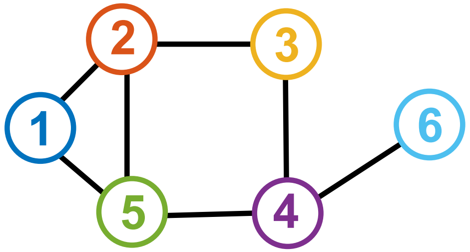


$$\mathbf{f}_i\left(\mathbf{x},u\right)=\left[\begin{array}{c} x_{(1)}\left(1-x_{(1)}^2-x_{(2)}^2\right)-\Omega_i x_{(2)}+u \\ x_{(2)}\left(1-x_{(1)}^2-x_{(2)}^2\right)+\Omega_i x_{(1)} \end{array}\right]$$

$$\mathbf{G}\left(\mathbf{y},\mathbf{x}\right)=\left[\begin{array}{c} 2\left(y_{(1)}-x_{(1)}\right) \\ 0 \end{array}\right] \Rightarrow h\left(\chi\right)=\sin\left(\chi\right)$$

$$r=\frac{1}{6}\sum_{i=1}^{6}\exp\left[\mathrm{i}\psi_i\right]$$

Network of 6 Stuart-Landau oscillators



$$\mathbf{f}_i(\mathbf{x}, u) = \begin{bmatrix} x_{(1)}\left(1 - x_{(1)}^2 - x_{(2)}^2\right) - \Omega_i x_{(2)} + u \\ x_{(2)}\left(1 - x_{(1)}^2 - x_{(2)}^2\right) + \Omega_i x_{(1)} \end{bmatrix}$$

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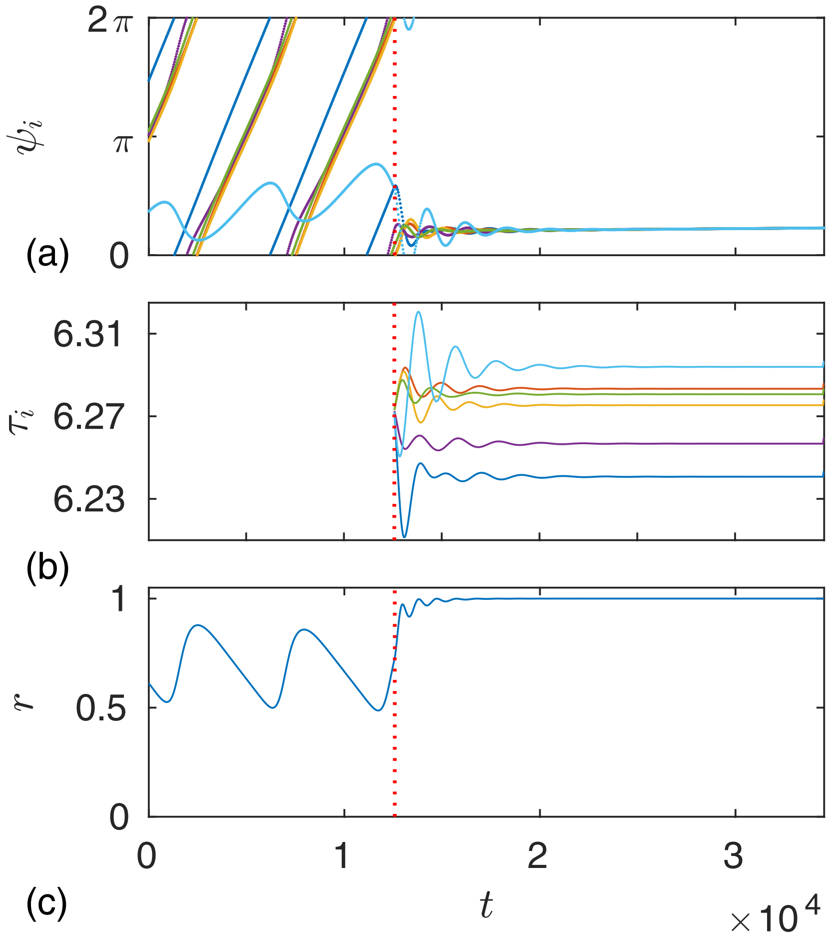
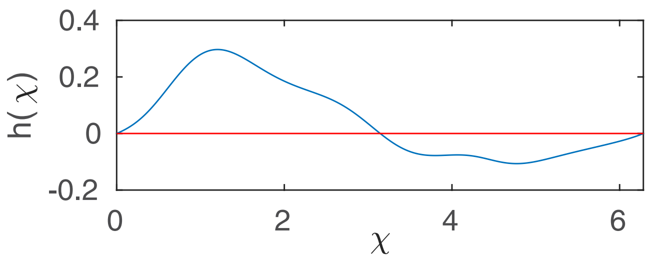


Network of 6 FitzHugh-Nagumo oscillators

$$\mathbf{f}_i(\mathbf{x}, u) = \begin{bmatrix} x_{(1)} - x_{(1)}^3/3 - x_{(2)} + 0.5 \\ \epsilon_i \left(x_{(1)} (1+u) + 0.7 - 0.8 x_{(2)} \right) \end{bmatrix}$$

$$\mathbf{G}(\mathbf{y}, \mathbf{x}) = \begin{bmatrix} y_{(1)} / \left(2 + y_{(2)}\right) - x_{(1)} / \left(2 + x_{(2)}\right) \\ 0 \end{bmatrix} \Rightarrow$$



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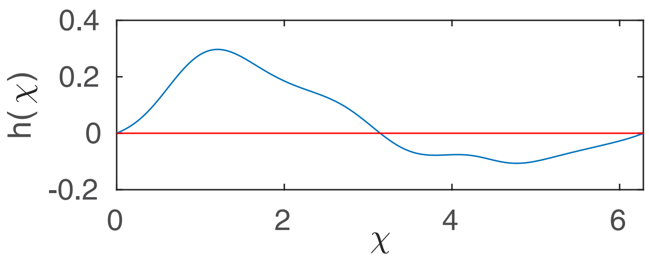

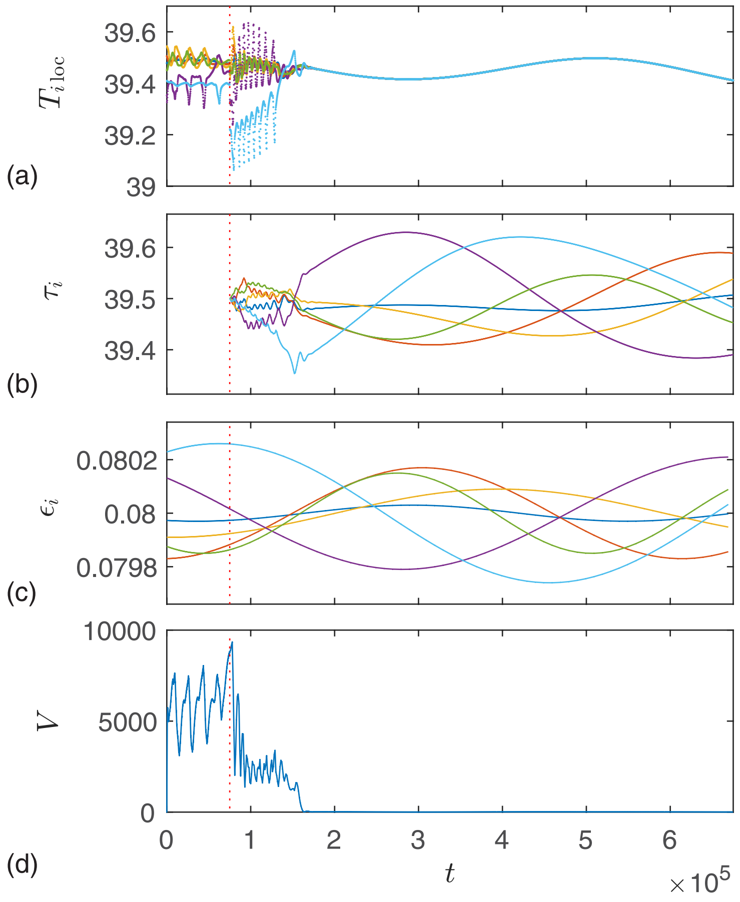

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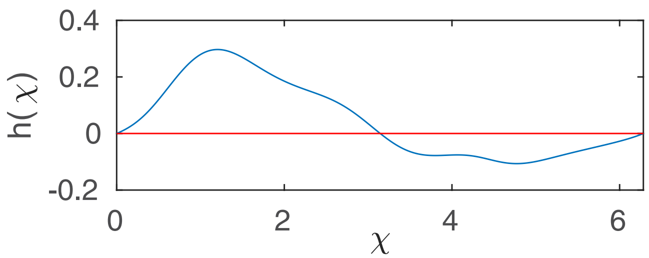

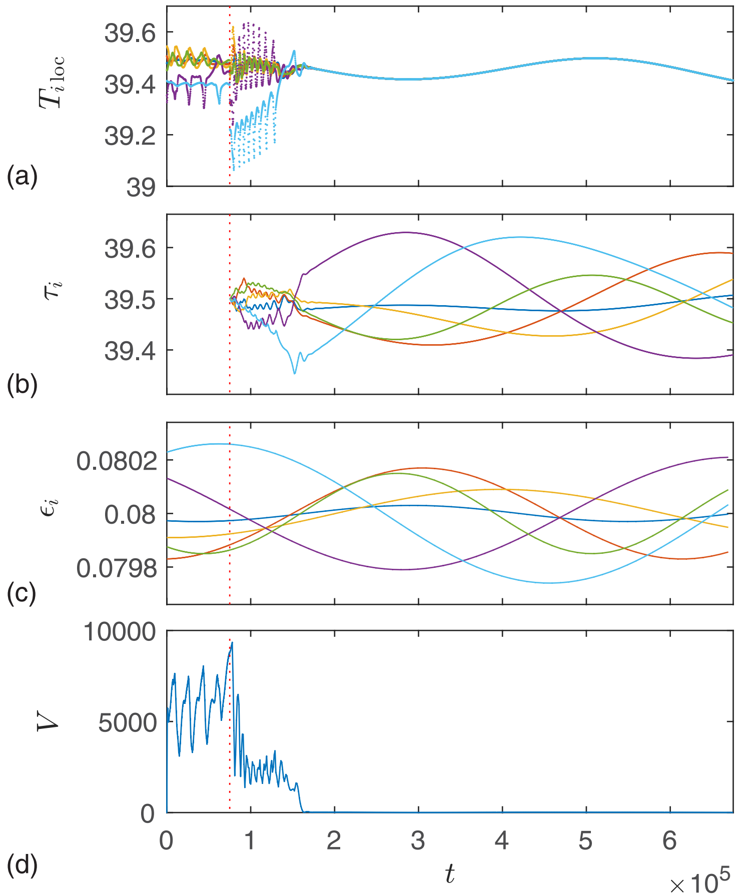

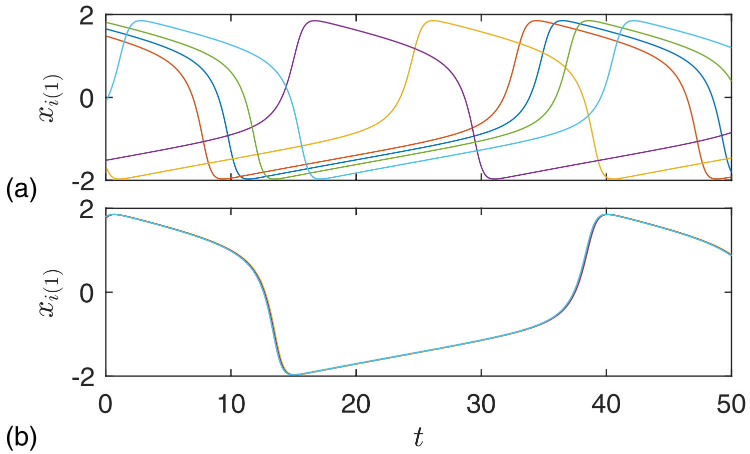

The end