Stationary Light and its Coherent Manipulation

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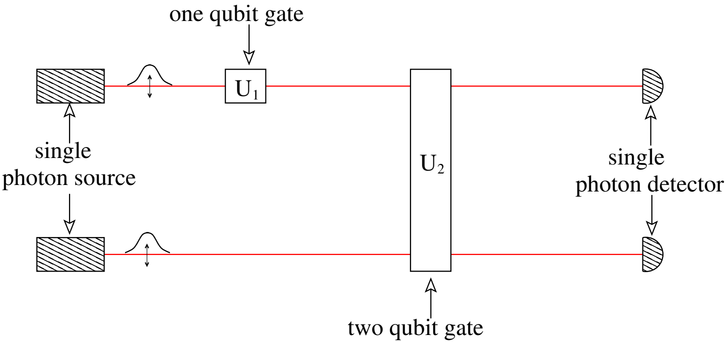
Contents

Contents

Quamntum computing

- based on:
  - photons as "flying" qubits
  - atomic ensembles as memory and processing units
- principle setup



Quantum gates

Principle Goal:

Construction of a high fidelity 2-qubit gate to entangle single photon pairs

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Controlled-phase gate

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|01\rangle &\longrightarrow |01\rangle \qquad & |11\rangle &\longrightarrow e^{i\varphi}|11\rangle
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- no or little absorption
- large nonlinear susceptibility

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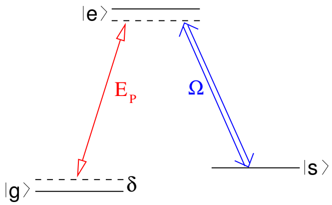
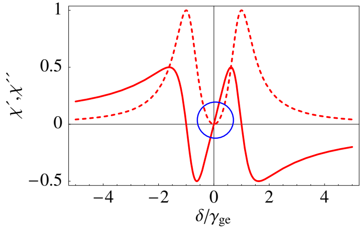
Problem:

photon-photon interaction negligible in generic media

Solution EIT und XPM





- no absorption at 2-photon resonance
- large dispersion ⇒ small group velocity
- second field: AC-Stark shift

  ⇒ phase shift Φ_{XPM} for first field

Large phase shift requires

- long interaction time ⇒ stationary light
- large intensity ⇒ compression of stationray light

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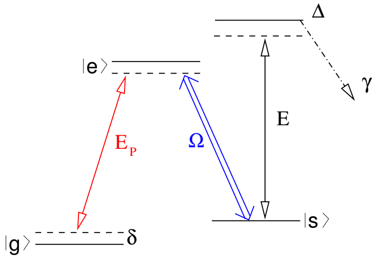

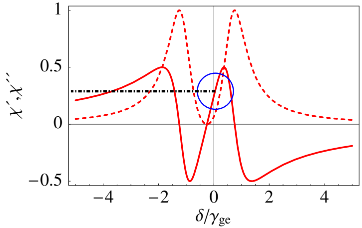

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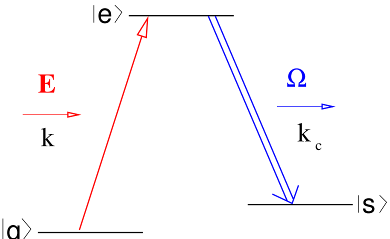

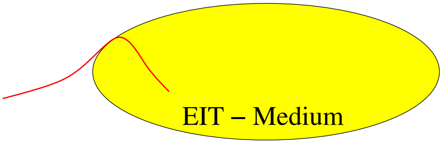

Dark- and Bright-state polaritons

$$\hat{\Psi} = \cos\theta \hat{E} - \sin\theta\sqrt{N}\hat{\sigma}_{gs} \quad \text{mit} \quad \tan^2\theta = g^2 N/\Omega^2$$

$$\hat{\Phi} = \sin\theta \hat{E} + \cos\theta\sqrt{N}\hat{\sigma}_{gs}$$

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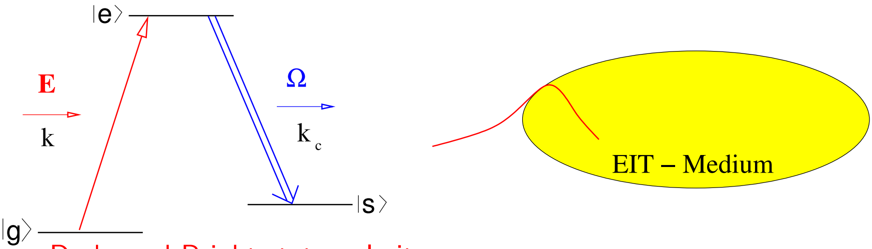

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form- and state-preserving dynamics

$$(\partial_t + c\cos^2\theta \partial_z)\hat{\Psi} = 0 \quad \text{und} \quad \hat{\Phi} \approx 0$$

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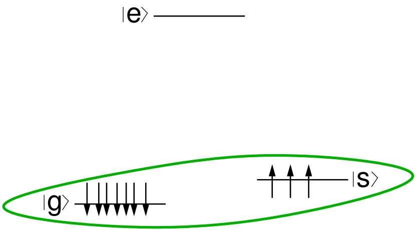

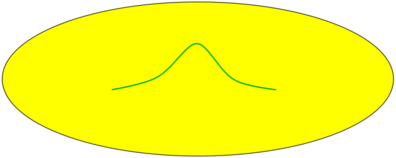

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adiabatic rotation

$$\Omega^2 \ll g^2 N \rightarrow \theta \approx \pi/2 \Longrightarrow \hat{\Psi} \approx -\sqrt{N}\hat{\sigma}_{gs}$$

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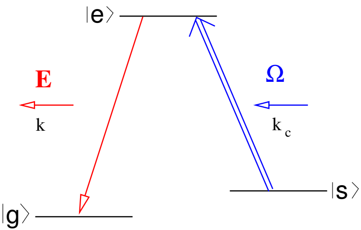

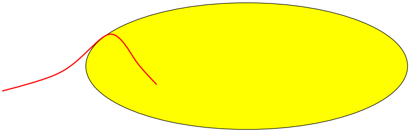

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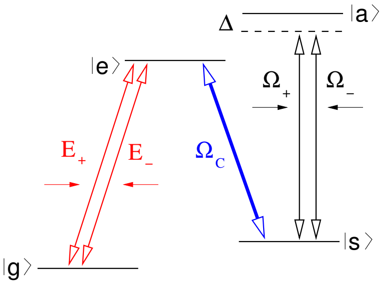
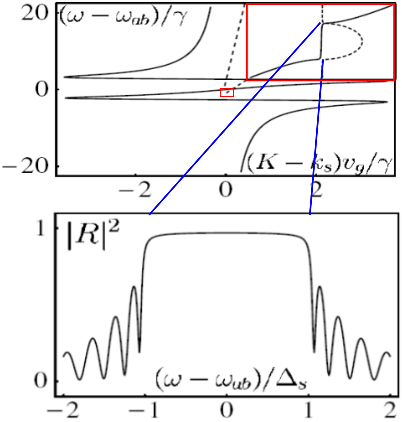
$$\Omega^2 \gg g^2 N \rightarrow \theta \approx 0 \Longrightarrow \hat{\Psi} \approx \hat{E}$$

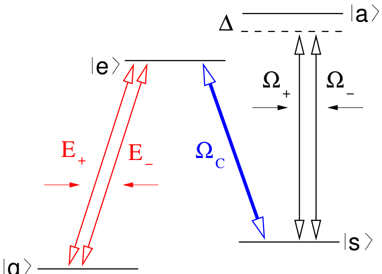
Photonic Band Gap Approach

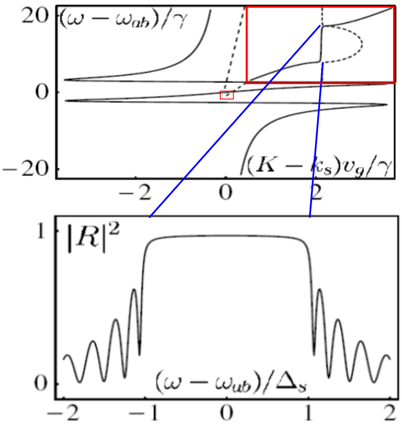




- spatial periodic modulation of n
- designable photonic band structure
- creation of standing wave in photonic band gap

Photonic Band Gap Approach

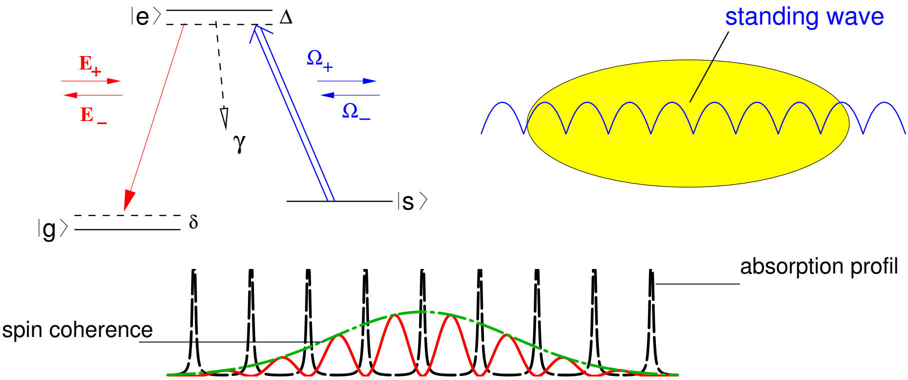




Conclusion

Stationary Light due to Bragg reflection from refractive index grating

Absorption Grating Approach



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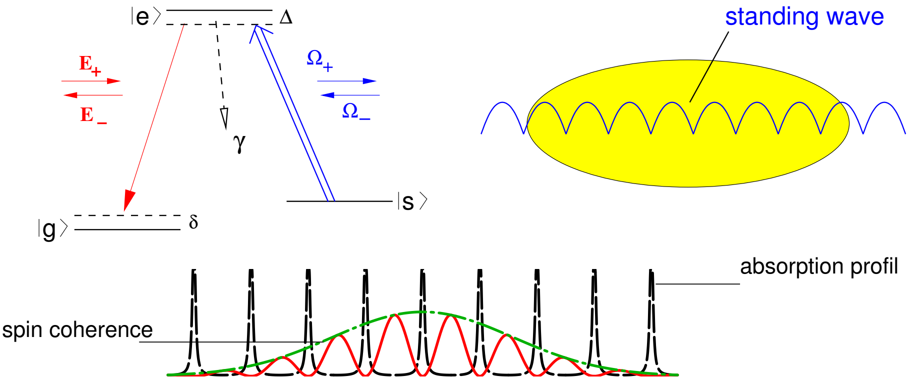

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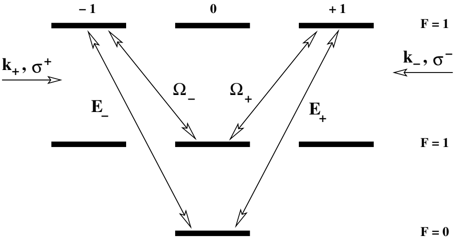
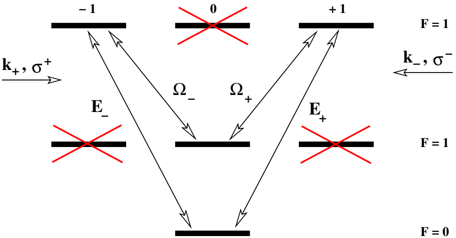


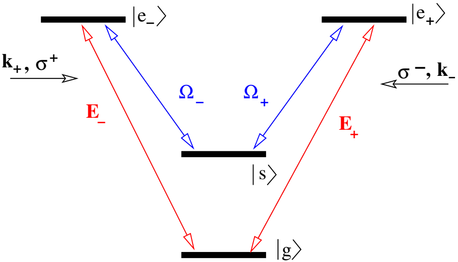
- field couple different transitions

4-Wave Mixing Approach



- field couple different transitions
- system can be simplified

4-Wave Mixing Approach



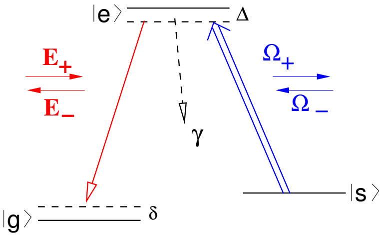
- no refrective index or absorption grating needed
- due to construction further external manipulation possible

Conjecture

4-wave mixing leads to Stationary Light

Contents

Equations of motion



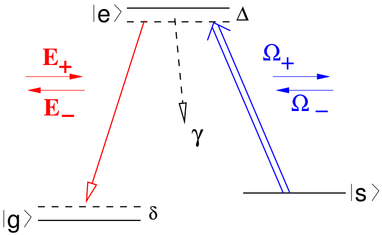
Heisenberg-Langevin equations

$$\frac{\mathrm{d}\hat{\sigma}_{\mu\nu}}{\mathrm{d}t} = -\gamma_{\mu\nu}\hat{\sigma}_{\mu\nu} - \frac{\mathrm{i}}{\hbar}\left[\hat{\sigma}_{\mu\nu}, \hat{H}\right] + \hat{F}_{\mu\nu}$$

with $\quad E(\mathbf{r}, t) = E_+ e^{\mathrm{i}kz} + E_- e^{-\mathrm{i}kz}$

- secular approximation
- weak probe-field limit
- subsituted into shortened wave equation

Equations of motion



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- secular approximation
- weak probe-field limit
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$\Rightarrow$ self-consistend system of field equations

Normal Modes

- Sum- und difference mode

$$E_S \equiv \cos\phi\, E_+ + \sin\phi\, E_-, \qquad E_D \equiv \sin\phi\, E_+ - \cos\phi\, E_-,$$

with $\tan\phi = \Omega_- / \Omega_+$

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- field equations in new basis

$$\begin{aligned}\left[\partial_t + v_{\rm gr}\cos(2\phi)\partial_z\right] E_S + v_{\rm gr}\sin(2\phi)\partial_z E_D & \\ -v_{\rm gr}\left[\partial_z\phi\right]\left[\sin(2\phi)E_S - \cos(2\phi)E_D\right] &= 0\end{aligned}$$

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whereas $v_{gr} = c\cos^2\theta$

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field equation can be manipulated using external control-fields

Pulse Matching

- E_D being absorbed $\Rightarrow$ $E_D \to 0$

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- known for EIT-systems
  S. E. Harris , Phys. Rev. Lett. **70**, 552, (1993)

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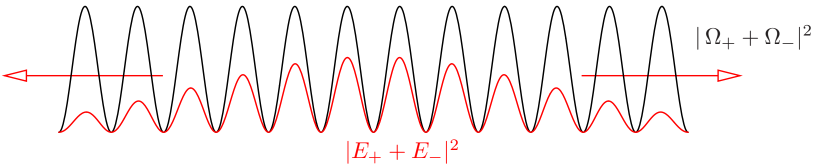

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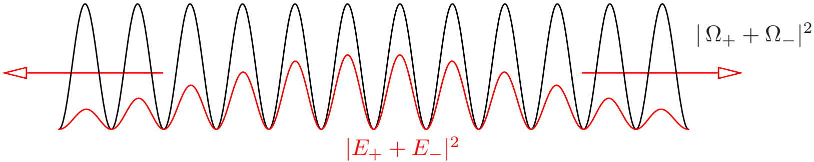

Conjecture

diffusive behavior of stationary light

1. equal amplitudes of control-fields

- $\Omega_+ = \Omega_- \Rightarrow \cot 2\phi = 0$
- adiabatic elimination of E_D
  for $T_D\, c/L_{\text{abs}} \gg 1$

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Diffusion equation

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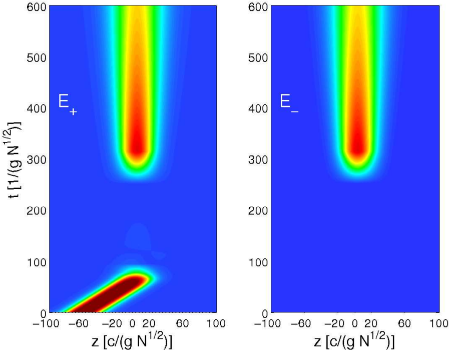


2. non-equal control-field amplitudes

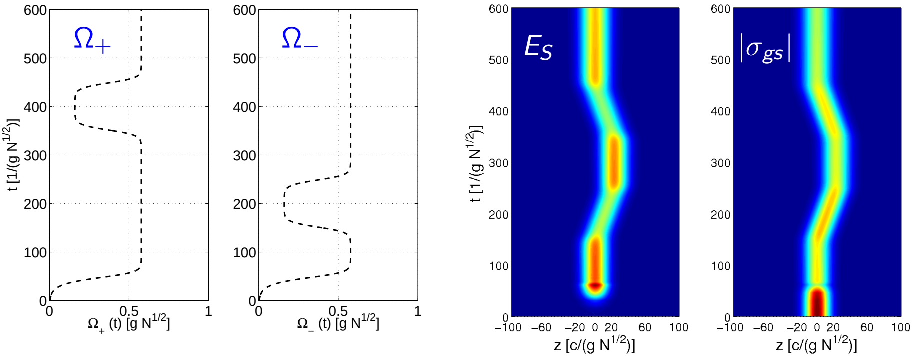
- adiabatic elimination leads to

Fokker-Planck equation

$$\left[\partial_t + v_{\mathrm{gr}} \cos(2\phi)\partial_z\right] E_S = v_{\mathrm{gr}}\, L_{\mathrm{abs}} \sin(2\phi)\partial_z^2 E_S$$

(a) diffusion term $v_{\mathrm{gr}}\, L_{\mathrm{abs}}\, \sin(2\phi)$
(b) drift term $v_{\mathrm{gr}}\, \cos(2\phi)$

2. non-equal control-field amplitudes



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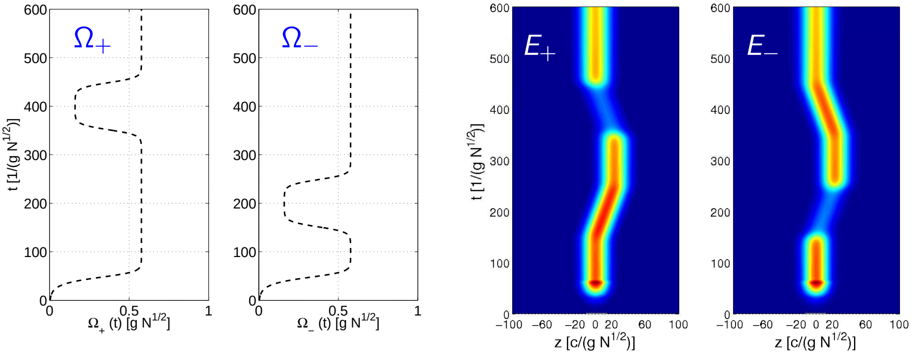
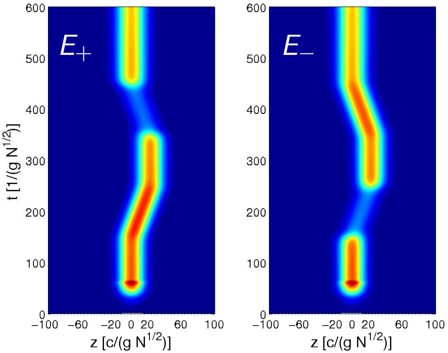


2. non-equal control-field amplitudes

Conclusion

Generation of probe-field stronger into direction of stronger control-field



Field Configuration for Compression

Problem

Look for field-configuration which leads to drift term that allows compression of e.m. excitation.

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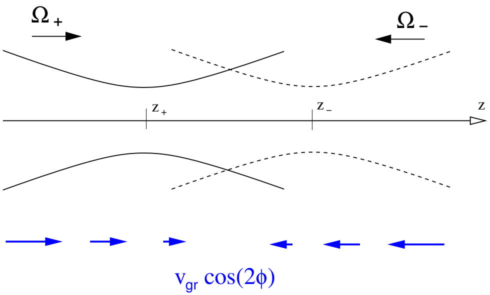

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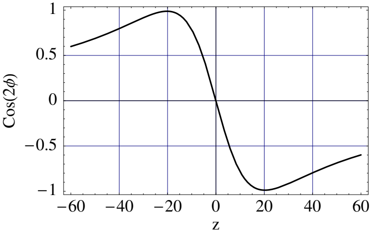

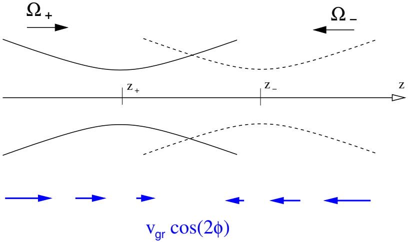

$$\cos(2\phi) \approx -\frac{z}{L} \qquad \& \qquad \sin(2\phi) \approx 1$$

Ornstein-Uhlenbeck Process

Fokker-Planck equation

$$\frac{\partial}{\partial t} E_S = -\frac{v_{\mathrm{gr}}}{L} E_S + \frac{\partial}{\partial z}\left[\frac{v_{\mathrm{gr}} z}{L} E_S\right] + \frac{\partial^2}{\partial z^2}\left[v_{\mathrm{gr}} L_{\mathrm{abs}} E_S\right]$$

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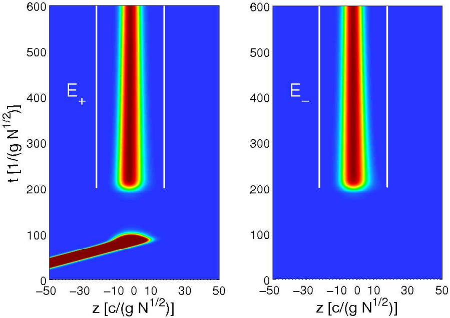
General Solution

$$E_S(z,t) = \frac{1}{\sqrt{2\,L\,L_{\mathrm{abs}}}} \exp\left\{-\frac{z^2}{2LL_{\mathrm{abs}}}\right\} \exp\left\{-\frac{v_{\mathrm{gr}}}{2L} t\right\} \times$$
$$\times \sum_{n=0}^{\infty} \frac{c_n}{\sqrt{2^n n! \sqrt{\pi}}} H_n\left(\frac{z}{\sqrt{2L\,L_{\mathrm{abs}}}}\right) \exp\left\{-n\frac{v_{\mathrm{gr}}}{L} t\right\}$$

Mode Matching

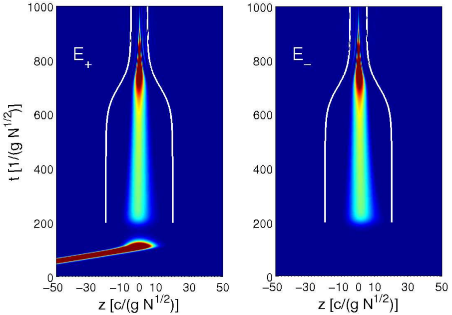
- to suppress absorption choose initial width $\Delta z \backsimeq \sqrt{L\,L_{\mathrm{abs}}}$

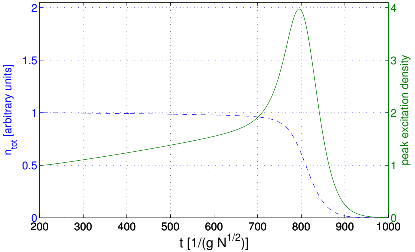


- mode matching $\Rightarrow$ $c_n \approx 0$ für $n \geq 1$ $\Rightarrow$ less absorption
- balance of diffusion and effective drift-force

Spatial Compression

- field compression using (adiabatic) reduction of foci distance





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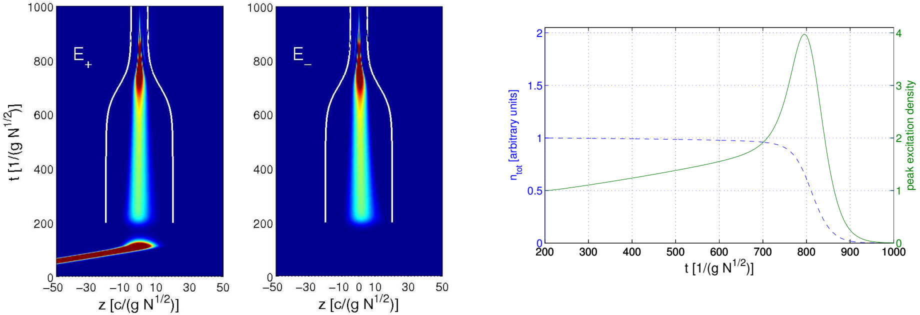

- large losses due to $\gamma_{\text{eff}} = v_{\text{gr}}/2L(t) \nearrow$ for $L(t) \searrow$

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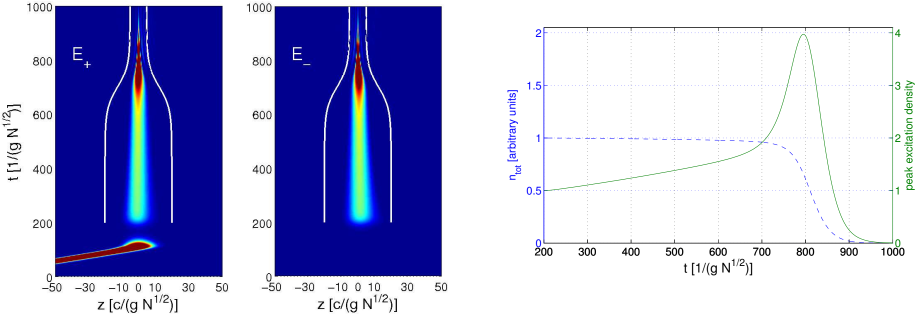


- large losses due to $\gamma_{\rm eff} = v_{\rm gr}/2L(t) \nearrow$ for $L(t) \searrow$
- nonadiabatic compression (?)

Nonadiabatic corrections

Corrections negligible for

- $|\partial_t \ln L| \ll c/L_{\mathrm{abs}}$
- $|z| \ll \left[\frac{L_{\mathrm{abs}}}{2\,L}\left(\frac{1}{c}\partial_t \ln L\right)^2\right]^{-1/2}$

Summarizing

Nonadiabatic correction negligible in general!

$$\Downarrow$$

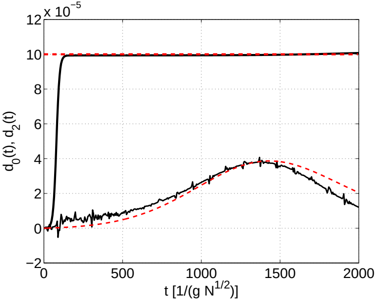
Losses only due to excitation of higher modes c_n, $n \geqq 1$

Losses due to excitation of higher modes

- two lowest expansion coefficients

$$\partial_t d_0 = 0 \quad \text{Konstante der Bewegung}$$
$$\partial_t d_2 = -2\left[\frac{v_{\mathrm{gr}}}{L} + \frac{1}{2}(\partial_t \ln L)\right] d_2 - \frac{1}{\sqrt{2}}(\partial_t \ln L)\, d_0$$

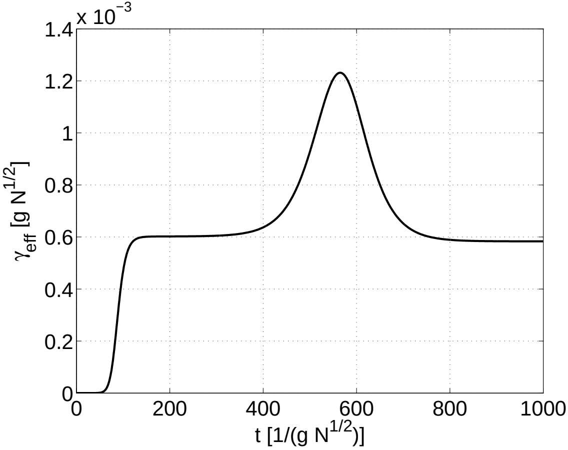
- Fall: $\left|\frac{v_{\mathrm{gr}}}{L}\right| > \left|\frac{1}{2}(\partial_t \ln L)\right|$



- good agreement
- because of $\partial_t \ln L < 0 \Rightarrow$ excitation of higher modes

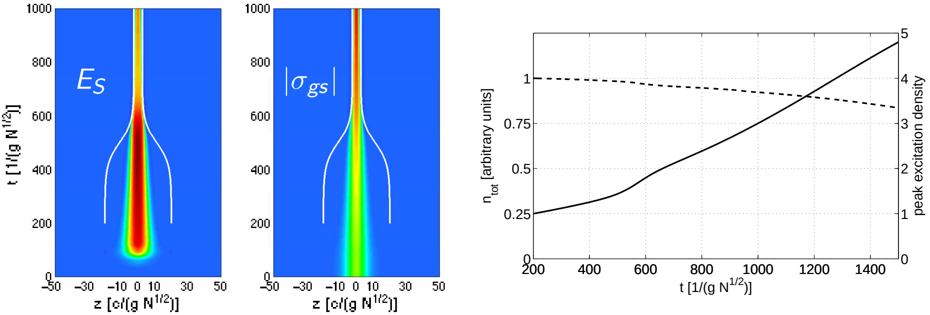
Spatial Compression - reviewed

- loss suppression due to temporally constant $\gamma_{\text{eff}} = \frac{v_{\text{gr}}(t)}{2L(t)}$



Spatial Compression - reviewed

- loss suppression due to temporally constant $\gamma_{\text{eff}} = \frac{v_{\text{gr}}(t)}{2L(t)}$



Contents

Résumé

- mode matching $\Rightarrow$ diffusion
- usage of special control-field configuration leads to
  (a) suppression of diffusive spreading
  (b) spatial compression of e.m. excitation
- studied decay channels
  (a) nonadiabatic dynamics negligible
  (b) excitation of higher Ornstein-Uhlenbeck modes leads to losses