Introduction to negative refraction

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introduction

- James Clerk Maxwell 1831-1879
  foundations of macroscopic electrodynamics

$$\left(\Delta - \varepsilon\mu \frac{1}{c^2}\partial_t^2\right)\mathbf{E} = \mathbf{0}$$

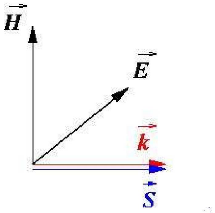
$$\mathbf{D} = \varepsilon\mathbf{E} \qquad \mathbf{B} = \mu\mathbf{H}$$

  - $\varepsilon > 0, \mu > 0 \Rightarrow$ propagating wave
  - $\varepsilon < 0, \mu > 0 \Rightarrow$ evanescent wave

- Victor Veselago 1968
  solutions for $\varepsilon < 0, \mu < 0$?

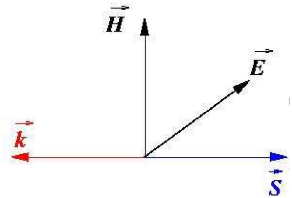
$$\mathbf{k} \times \mathbf{E} = \mu \frac{\omega}{c} \mathbf{H}$$
$$\mathbf{k} \times \mathbf{H} = - \varepsilon \frac{\omega}{c} \mathbf{E}$$

$\varepsilon > 0, \mu > 0$



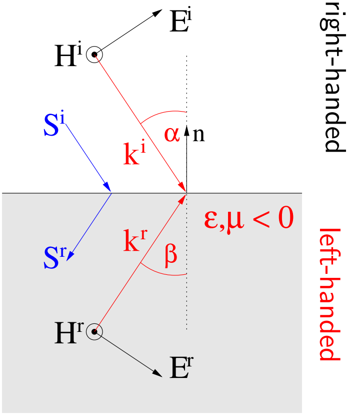
right-handed

$\varepsilon < 0, \mu < 0$

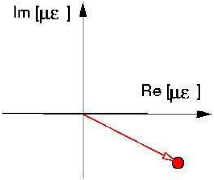


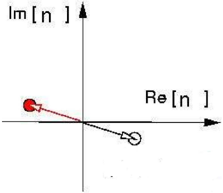
left-handed

- apply boundary conditions
- outgoing wave on the negative side
- **E**, **H**, **k** left-handed
  due to conservation of momentum along interface
- energy conservation ok



$$n^2 = \varepsilon \cdot \mu$$





$$\Rightarrow n(-1,-1) = -1$$

- field energy

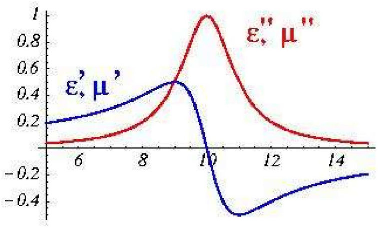
$$w = \text{Re}\left[\frac{d(\omega\varepsilon)}{d\omega}\right] \mathbf{E}^2 + \text{Re}\left[\frac{d(\omega\mu)}{d\omega}\right] \mathbf{H}^2 > 0$$

$$\Rightarrow \quad \text{Re}\varepsilon + \omega\frac{d\text{Re}\varepsilon}{d\omega} > 0 \qquad \text{Re}\mu + \omega\frac{d\text{Re}\mu}{d\omega} > 0$$

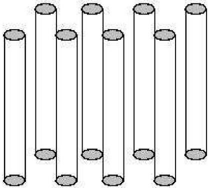
- dispersion is essential

  $\Rightarrow \varepsilon(\omega), \mu(\omega)$ complex

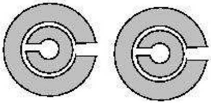
- structure $\ll$ wavelength
  $\Rightarrow$ suitable for microwaves



elektrische Dipole:



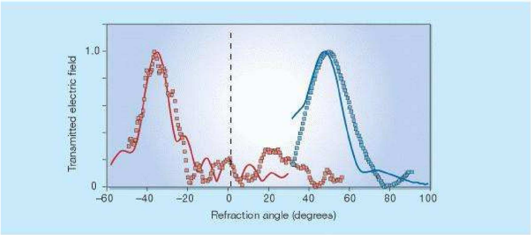
magnetische Dipole:

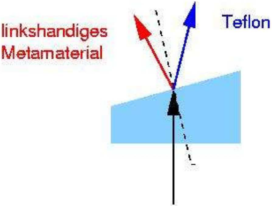


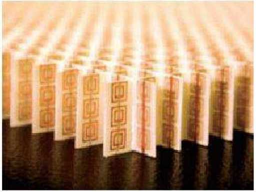
- control of size $\Rightarrow$ control of resonance frequency

experimental proof for $n < 0$

Shelby, Smith, Schultz, Science **292**,77 (2001)







experimental progress

- Yen *et. al.* Science (2004): few THz
- Linden *et. al.* Science (2004): 100 THz
- Zhang *et. al.* PRL (2005): 230 THz (1,3 μm)
- Katsarakis *et. al.* Optics Letters (2005): few THz 5 layers

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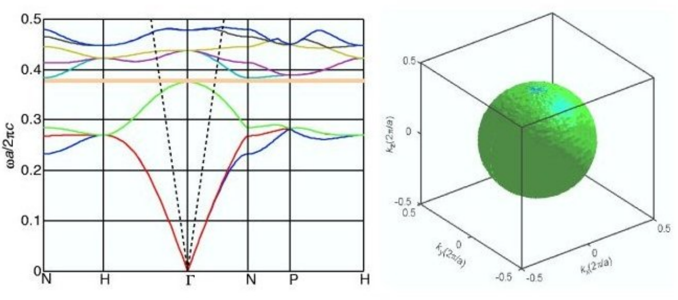
But:

resonance frequencies of LHM saturate at $\approx$ 300 THz

photonic crystals

- structure size ≈ wavelength
- band structure (numerical data)



- second band has a (almost) spherical constant-frequency surface and a negative effective photon-mass

⇒ negative effective refractive index

photonic crystals

- Parimi *et. al.* PRL (2003): microwaves
- Berrier *et. al.* PRL (2004): infrared
- Lu *et. al.* PRL (2005): microwaves 3D

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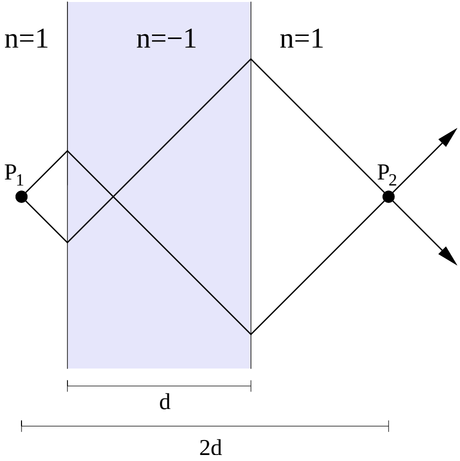
But:

technologically hard to realize

flat lens

Veselago Sov. Phys. Usp. **10**, 509 (1968)



phase difference for

propagating modes is zero

perfect lens

- Pendry PRL **85**, 3966 (2000)
- resolution of points with $\Delta x < \lambda$ needs $k_\perp > \frac{2\pi}{\lambda}$
  $\Rightarrow$ evanescent waves $k_z = i\sqrt{k_\perp^2 - \frac{\omega^2}{c^2}}$
- $k_z' = -k_z$
  $\Rightarrow$ overall amplitude of evanescent modes: $e^{-k_z d} e^{+k_z d} = 1$
- perfect imaging properties
  $\Rightarrow$ space with thickness $2d$ has optical length $l_{\text{opt}} = 0$

restrictions

- perfect lens demands perfect impedance matching
- resolution Δx

$$\Delta n = \exp\left\{-\frac{\Delta x}{2\pi d}\right\}$$

- exponentially more restrictive with increasing thickness

Smith *et. al.* Appl. Phys. Lett. **82**, 1506 (2003)

Merlin Appl. Phys. Lett **84**, 1290 (2004)

specifications

- perfect lens
  - subwavelength resolution: evanescent modes
  - impedance matching condition

need a tunable low-absorption negative index material

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solution: coherently prepared media?

LHM and EIT

idea

couple electric and magnetic atomic transitions to induce magnetisation

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couple electric and magnetic atomic transitions to induce magnetisation

- Oktel, Müstecaplioglu PRA **70**, 053806 (2004)
- Thommen, Mandel PRL **96**, 053601 (2006)
  - **B**-field not taken into account
  - no local-field corrections
  - densities: $10^{18}\,\frac{1}{m^3} - 10^{19}\,\frac{1}{m^3}$
  - large absorption: $\frac{\mathrm{Im}\,n}{\mathrm{Re}\,n} \approx 1$

A new concept: usage of chirality

chiral medium proposed by Sir John Pendry (Science **306**, 1353)

$$\mathbf{P} = \chi_e \mathbf{E} + \xi_{EH} \mathbf{H},$$
$$\mathbf{M} = \xi_{HE} \mathbf{E} + \chi_m \mathbf{H}$$

- index of refraction is altered to

$$n = \sqrt{\varepsilon\mu - \frac{(\xi_{EH} + \xi_{HE})^2}{4}} + \frac{i}{2}(\xi_{EH} - \xi_{HE})$$

- choose imaginary chirality: $\xi_{EH} = i\xi = -\xi_{HE}$

$$\boxed{n = \sqrt{\varepsilon\mu} - \xi}$$

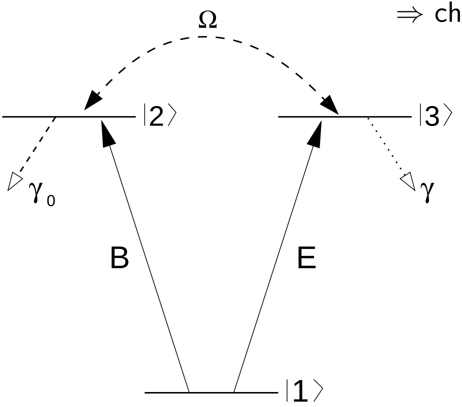
Why is there a gain?

- $\frac{\mathbf{d}_{mag}^2}{\mathbf{d}_{elec}^2} \approx \alpha^2 \approx \frac{1}{137^2}$
- for inhomogeneously broadened media:
  - magnetic response small: $\boxed{\chi_m \approx \alpha^2 \chi_e}$

  $\Rightarrow$ very hard to get $\mu = 1 + 4\pi\chi_e < 0$
  - In contrast: $\boxed{\xi_i \approx \alpha \chi_e}$
- But: absorption

V-type system

- $\mathbf{E}, \mathbf{B}$ electric/magnetic part of probe field
- Ω cross couples electric and magnetic transition
  $\Rightarrow$ chiral behaviour



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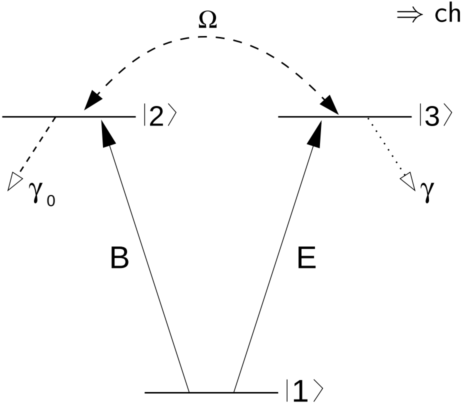
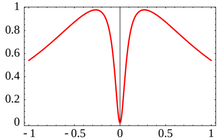


- $\gamma_0 \ll \gamma \Rightarrow$ EIT



χ_e

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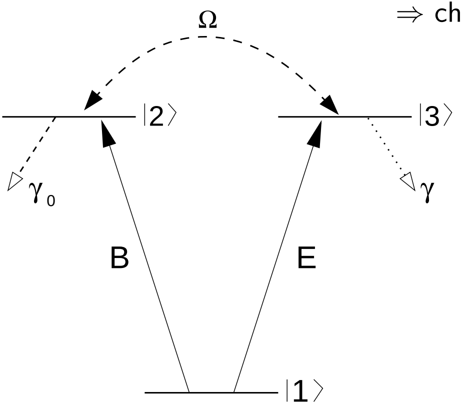

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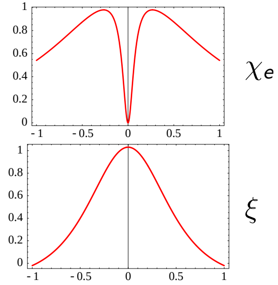

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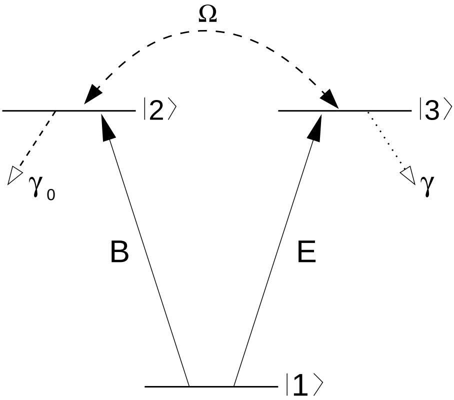
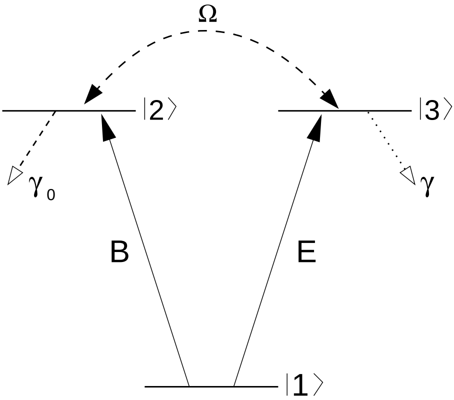


- ☹ Ω: *dc*-coupling $\Rightarrow$ phase of ξ not free to set

V-type system



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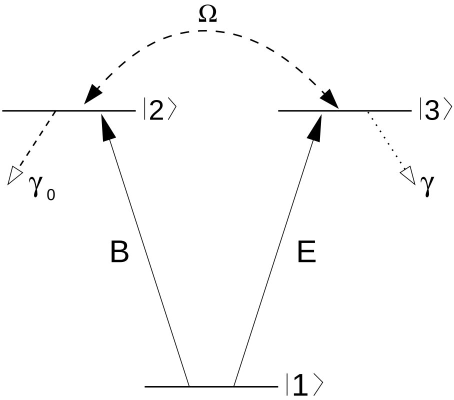
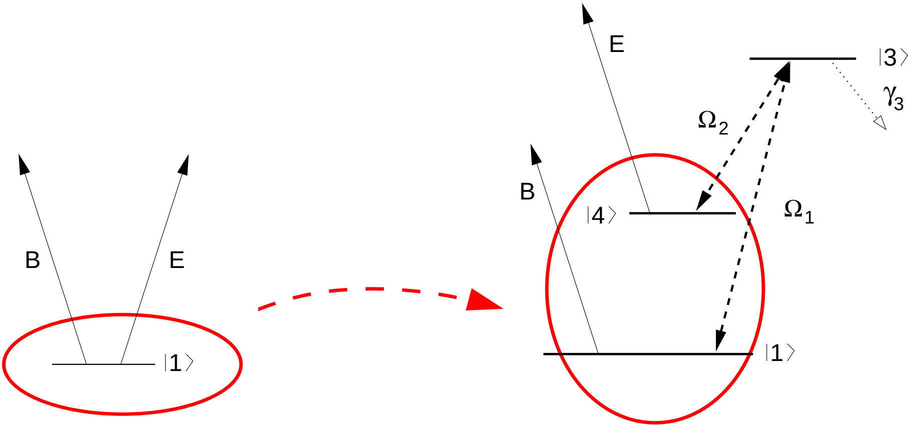
- ☹ no EIT for inhomogeneously broadened systems

V-type system



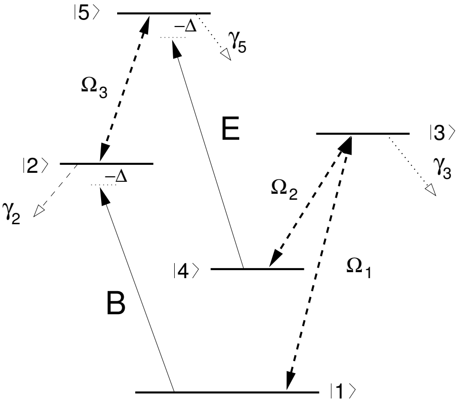
- ☹ Ω: *dc*-coupling ⇒ phase of ξ not free to set
- ☹ no EIT for inhomogeneously broadened systems
- ☹ level scheme not found in real atoms

solution

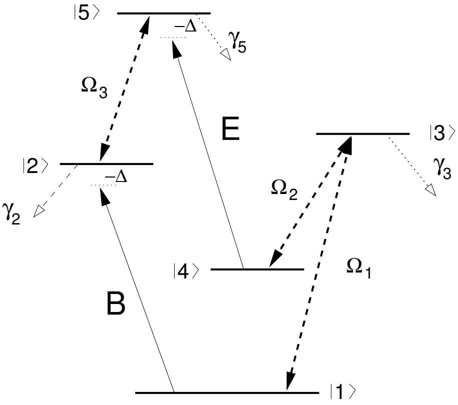


$$|D\rangle \approx \Omega_2\,|1\rangle - \Omega_1\,|4\rangle$$

5-level system

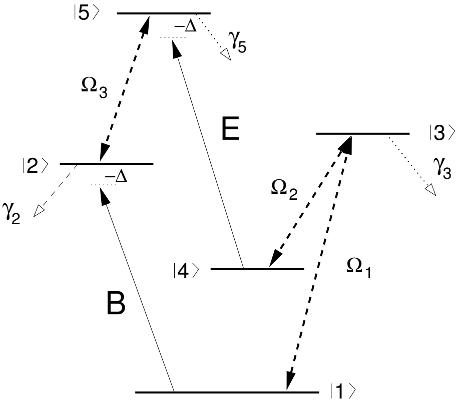


5-level system



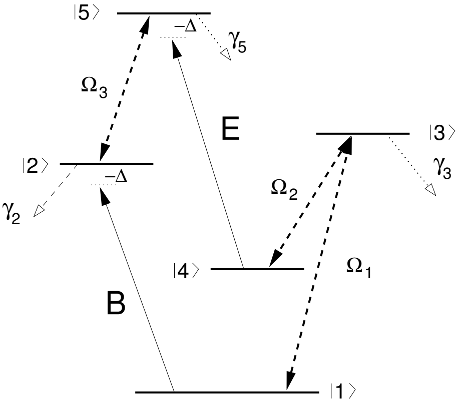
- 🙂 much easier to realize

5-level system



- ☺ much easier to realize
- ☺ Ω_3: *ac*-coupling ⇒ phase of cross coupling ξ can be set

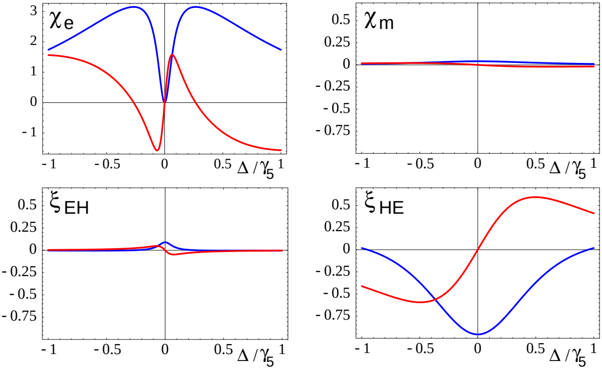
5-level system



- much easier to realize
- Ω_3: *ac*-coupling ⇒ phase of cross coupling ξ can be set
- $E_2 \approx E_4$: EIT also for inhomogeneous broadening

resulting susceptibilities [arb. u.]

- inhomogeneous broadening $\gamma_P \approx$ dielectric linewidth γ_5

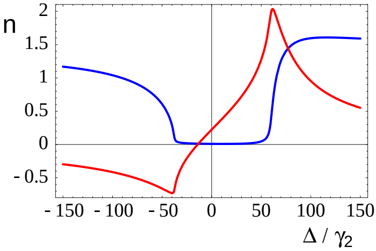


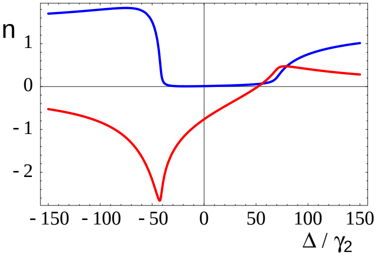
real, imaginary part

refractive index

- local field corrections included

$N = 5 \cdot 10^{16} \frac{1}{cm^3}$ $\qquad$ $N = 5 \cdot 10^{17} \frac{1}{cm^3}$

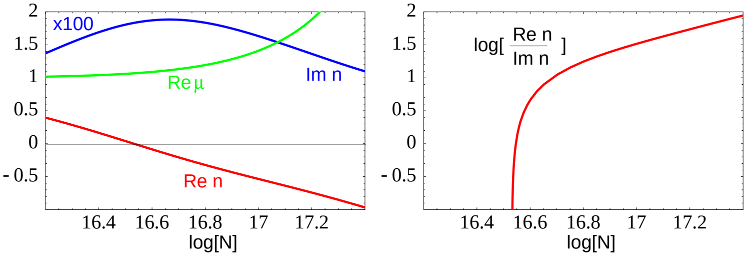




real, imaginary part

refraction/absorption ratio

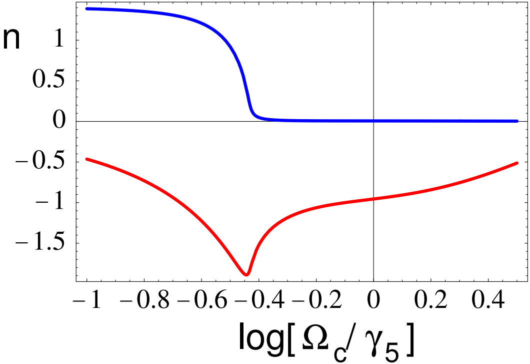
- local field corrections included



very high refraction/absorption ratio

tunability

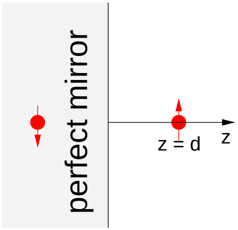
- perfect lens very sensitive to impedance mismatch

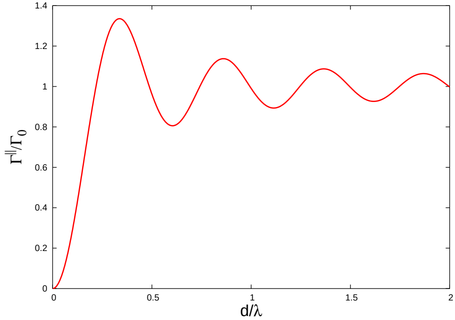


value of *n* fine-tunable

Purcell-effect in front of a mirror

- mirror generates induced dipole
- boundary conditions ⇒ component $\parallel$ to mirror is 180° out of phase

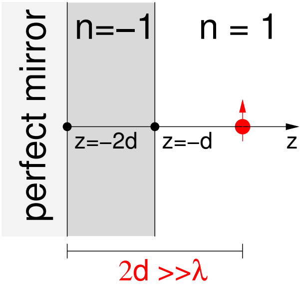




mirror + perfect lens

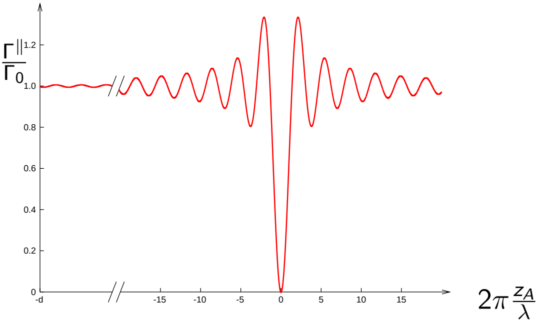
- $\Gamma = \frac{2\omega_A^2 d_i d_j}{\hbar\varepsilon_0 c^2} \mathrm{Im}\left[G_{ij}(\mathbf{r}_A, \mathbf{r}_A, \omega_A)\right]$

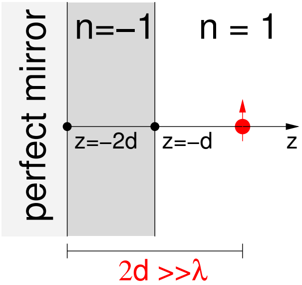
PRA **68**, 043816 (2003)



mirror + perfect lens

- analytical expression for the Greensfunction





$\Rightarrow$ translation of the Purcell-effect over distance $2d$

J.K. & M. Fleischhauer PRA **71**, 011804 (2005), Las. Phys. **15**, 1 (2005)