Local field effects in negative-index materials

Jürgen Kästel

AG Quantenoptik
Fachbereich Physik
Technische Universität Kaiserslautern

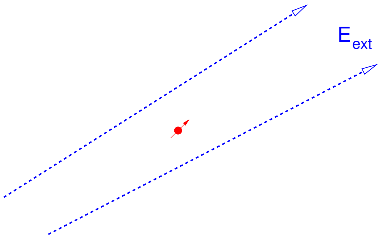
1. Motivation
2. microscopic theory
   - dielectric media
   - magnetic media

1. Motivation

2. microscopic theory
   - dielectric media
   - magnetic media

local fields for a dielectric

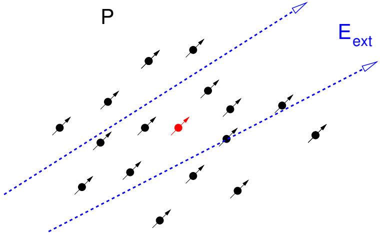
- polarization $\mathbf{P} = N\alpha\mathbf{E}_{\text{loc}}$
- electric dipole feels
  - external electric field $\mathbf{E}_{\text{ext}}$
- $\varepsilon = 1 + 4\pi N\alpha$



local fields for a dielectric

- polarization $\mathbf{P} = N\alpha\mathbf{E}_{\text{loc}}$
- electric dipole feels
  - external electric field $\mathbf{E}_{\text{ext}}$
  - polarized medium

  $\Rightarrow$ $\mathbf{E}_{\text{loc}} = \mathbf{E}_{\text{ext}} + \frac{4\pi}{3}\mathbf{P}$
- $\varepsilon = 1 + \frac{4\pi N\alpha}{1-\frac{4\pi}{3}N\alpha}$



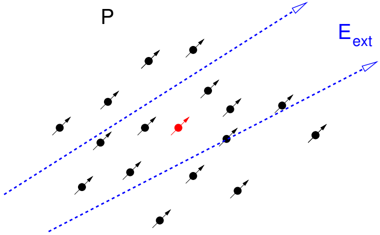
Surprisingly

The local field factor is quite accurate

local fields for a dielectric

- polarization $\mathbf{P} = N\alpha\mathbf{E}_{loc}$
- electric dipole feels
  - external electric field $\mathbf{E}_{ext}$
  - polarized medium

  $\Rightarrow$ $\mathbf{E}_{loc} = \mathbf{E}_{ext} + \frac{4\pi}{3}\mathbf{P}$



- $\varepsilon = 1 + \frac{4\pi N\alpha}{1 - \frac{4\pi}{3}N\alpha}$
- $\varepsilon \xrightarrow{N\to\infty} -2$

$\Rightarrow$ dielectrics get opaque

Surprisingly

The local field factor is quite accurate

local fields for a LHM

- local electric field
  - $\mathbf{E}_{\text{loc}} = \mathbf{E}_{\text{ext}} + \frac{4\pi}{3}\mathbf{P}$
  - $\varepsilon = 1 + \frac{4\pi N\alpha_e}{1-\frac{4\pi}{3}N\alpha_e}$
  - electric polarizability α_e

$$\Rightarrow \varepsilon \stackrel{N\to\infty}{\longrightarrow} -2$$

- local magnetic field
  - $\mathbf{H}_{\text{loc}} = \mathbf{H}_{\text{ext}} + \frac{4\pi}{3}\mathbf{M}$
  - $\mu = 1 + \frac{4\pi N\alpha_m}{1-\frac{4\pi}{3}N\alpha_m}$
  - magnetic polarizability α_m

$$\Rightarrow \mu \stackrel{N\to\infty}{\longrightarrow} -2$$

local fields for a LHM

- local electric field
  - $\mathbf{E}_{\text{loc}} = \mathbf{E}_{\text{ext}} + \frac{4\pi}{3}\mathbf{P}$
  - $\varepsilon = 1 + \frac{4\pi N\alpha_e}{1 - \frac{4\pi}{3}N\alpha_e}$
  - electric polarizability α_e

$$\Rightarrow \varepsilon \stackrel{N\to\infty}{\longrightarrow} -2$$

- local magnetic field
  - $\mathbf{H}_{\text{loc}} = \mathbf{H}_{\text{ext}} + \frac{4\pi}{3}\mathbf{M}$
  - $\mu = 1 + \frac{4\pi N\alpha_m}{1 - \frac{4\pi}{3}N\alpha_m}$
  - magnetic polarizability α_m

$$\Rightarrow \mu \stackrel{N\to\infty}{\longrightarrow} -2$$

theorem

$$n = -2$$

Increase density ⇒ get negative refraction

dielectrics

Vries, Coevorden, Lagendijk, Rev. Mod. Phys. **70**, 447 (1998)

- Helmholtz equation:

$$\nabla \times \nabla \times \mathbf{E} - \frac{\omega^2}{c^2}\mathbf{E} = i\omega\mu_0\mathbf{j} + \mu_0\omega^2\mathbf{P}$$

- Greens function: $\mathbf{P} = \alpha_e(\mathbf{r})\mathbf{E}$

$$\left[\frac{\omega^2}{c^2} - \nabla \times \nabla \times\right]\mathbf{G}(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}')\mathbb{1}_3 - \mu_0\omega^2\alpha_e(\mathbf{r})\mathbf{G}(\mathbf{r},\mathbf{r}')$$

- field solution

$$\mathbf{E}(\mathbf{r}) = -i\omega\mu_0\int d\mathbf{r}'\mathbf{G}(\mathbf{r},\mathbf{r}')\mathbf{j}(\mathbf{r}')$$

dielectrics, single scatterer

- Greens function

$$\left[\frac{\omega^2}{c^2} - \nabla \times \nabla \times\right] \mathbf{G}(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}')\mathbb{1}_3 + \mathcal{V}(\mathbf{r})\mathbf{G}(\mathbf{r}, \mathbf{r}')$$

- solution: Dyson equation

$$\mathbf{G} = \mathbf{G}_0 + \mathbf{G}_0 \mathcal{T} \mathbf{G}_0 \qquad \mathcal{T} = \mathcal{V} + \mathcal{V}\mathbf{G}_0\mathcal{V} + \dots$$

dielectrics, single scatterer

- Greens function

$$\left[\frac{\omega^2}{c^2} - \nabla \times \nabla \times\right] \mathbf{G}(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}')\mathbb{1}_3 + \mathcal{V}(\mathbf{r})\mathbf{G}(\mathbf{r}, \mathbf{r}')$$

- solution: Dyson equation

$$\mathbf{G} = \mathbf{G}_0 + \mathbf{G}_0 \mathcal{T} \mathbf{G}_0 \qquad \mathcal{T} = \mathcal{V} + \mathcal{V}\mathbf{G}_0\mathcal{V} + \dots$$

- e.g. single point scatterer: $\mathcal{V}(\mathbf{r}) = v\delta(\mathbf{r} - \mathbf{r}_0)\mathbb{1}_3$

$$\Rightarrow \mathcal{T} = \frac{1}{v - \mathbf{G}_0(\mathbf{r}=0)}$$

dielectrics, single scatterer

- Greens function

$$\left[\frac{\omega^2}{c^2} - \nabla \times \nabla \times\right] \mathbf{G}(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}')\mathbb{1}_3 + \mathcal{V}(\mathbf{r})\mathbf{G}(\mathbf{r}, \mathbf{r}')$$

- solution: Dyson equation

$$\mathbf{G} = \mathbf{G}_0 + \mathbf{G}_0 \mathcal{T} \mathbf{G}_0 \qquad \mathcal{T} = \mathcal{V} + \mathcal{V}\mathbf{G}_0\mathcal{V} + \dots$$

- e.g. single point scatterer: $\mathcal{V}(\mathbf{r}) = v\delta(\mathbf{r} - \mathbf{r}_0)\mathbb{1}_3$

$$\Rightarrow \mathcal{T} = \frac{1}{v - \mathbf{G}_0(\mathbf{r}=0)}$$

result

spontaneous decay rate Γ, Lamb shift $\Delta\nu$

dielectric medium

- single cubic lattice of point scatterers:

$$\mathcal{V}(\mathbf{r}) = v \sum_{\mathbf{R}} \delta(\mathbf{r} - \mathbf{R}) \mathbb{1}_3$$

R: lattice vector

- exact result:

$$\mathcal{T}(\mathbf{k}, \mathbf{k}') = \sum_{\mathbf{R}} e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{R}} \frac{1}{v - \sum_{\mathbf{R}} e^{i\mathbf{k}'\cdot\mathbf{R}} \mathbf{G}_0(\mathbf{R})}$$

density dependence via $\sum_{\mathbf{R}} e^{i\mathbf{k}\cdot\mathbf{R}} \mathbf{G}_0(\mathbf{R}) = \frac{1}{\Omega} \sum_{\mathbf{K}} \frac{1}{\frac{\omega^2}{c^2} - |\mathbf{k}-\mathbf{K}|^2 \Delta_{\mathbf{k}-\mathbf{K}}}$

result

local field correction $\varepsilon = 1 + \frac{4\pi N\alpha_e}{1 - \frac{4\pi}{3} N\alpha_e}$

magnetic media

- Helmholtz equation:

$$\nabla \times \nabla \times \mathbf{B} - \frac{\omega^2}{c^2}\mathbf{B} = \mu_0 \nabla \times \mathbf{j} + \mu_0 \nabla \times \nabla \times \mathbf{M}$$

- Greens function: $\mathbf{M} = \alpha_m(\mathbf{r})\mathbf{B}$

$$\left[\frac{\omega^2}{c^2} - \nabla \times \nabla \times\right] \mathbf{G}_m(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}') + \frac{1}{\omega^2} \nabla \times \nabla \times \mathcal{V}_m(\mathbf{r}) \mathbf{G}_m(\mathbf{r}, \mathbf{r}')$$

- field solution

$$\mathbf{B}(\mathbf{r}) = -\mu_0 \int d\mathbf{r}' \mathbf{G}_m(\mathbf{r}, \mathbf{r}') \nabla' \times \mathbf{j}(\mathbf{r}')$$

magnetic media

- Helmholtz equation:

$$\nabla \times \nabla \times \mathbf{B} - \frac{\omega^2}{c^2}\mathbf{B} = \mu_0 \nabla \times \mathbf{j} + \mu_0 \nabla \times \nabla \times \mathbf{M}$$

- Greens function: $\mathbf{M} = \alpha_m(\mathbf{r})\mathbf{B}$

$$\left[\frac{\omega^2}{c^2} - \nabla \times \nabla \times\right] \mathbf{G}_m(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}') + \frac{1}{\omega^2} \nabla \times \nabla \times \mathcal{V}_m(\mathbf{r}) \mathbf{G}_m(\mathbf{r}, \mathbf{r}')$$

- field solution

$$\mathbf{B}(\mathbf{r}) = -\mu_0 \int d\mathbf{r}' \mathbf{G}_m(\mathbf{r}, \mathbf{r}') \nabla' \times \mathbf{j}(\mathbf{r}')$$

problem

equation structure different: $\nabla \times \nabla \times \mathbf{M}$ not $\omega^2 \mathbf{M}$

magnetic media

- $\nabla \times \nabla \times \mathbf{G}_0 = \frac{\omega^2}{c^2}\mathbf{G}_0$ holds
- point particles: $\mathcal{T}_m$ exactly solvable

$$\mathcal{T}_m(\mathbf{k},\mathbf{k}') = \sum_{\mathbf{R}} e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{R}} \frac{1}{v_m - \sum_{\mathbf{R}} e^{i\mathbf{k}'\cdot\mathbf{R}}\mathbf{G}_0(\mathbf{R})}$$

magnetic media

- $\nabla \times \nabla \times \mathbf{G}_0 = \frac{\omega^2}{c^2}\mathbf{G}_0$ holds
- point particles: $\mathcal{T}_m$ exactly solvable

$$\mathcal{T}_m(\mathbf{k},\mathbf{k}') = \sum_{\mathbf{R}} e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{R}} \frac{1}{v_m - \sum_{\mathbf{R}} e^{i\mathbf{k}'\cdot\mathbf{R}}\mathbf{G}_0(\mathbf{R})}$$

result

local field correction $\mu = 1 + \frac{4\pi N\alpha_m}{1-\frac{4\pi}{3}N\alpha_m}$

negative refracting media

- coupled Helmholtz equations

$$\nabla \times \nabla \times \mathbf{E} - \frac{\omega^2}{c^2}\mathbf{E} = i\omega\mu_0\mathbf{j} + i\omega\mu_0\nabla \times \mathbf{M} + \mu_0\omega^2\mathbf{P}$$

$$\nabla \times \nabla \times \mathbf{B} - \frac{\omega^2}{c^2}\mathbf{B} = \mu_0\nabla \times \mathbf{j} + \mu_0\nabla \times \nabla \times \mathbf{M} - i\omega\mu_0\nabla \times \mathbf{P}$$

negative refracting media

- coupled Helmholtz equations

$$\nabla \times \nabla \times \mathbf{E} - \frac{\omega^2}{c^2}\mathbf{E} = i\omega\mu_0\mathbf{j} + i\omega\mu_0\nabla \times \mathbf{M} + \mu_0\omega^2\mathbf{P}$$

$$\nabla \times \nabla \times \mathbf{B} - \frac{\omega^2}{c^2}\mathbf{B} = \mu_0\nabla \times \mathbf{j} + \mu_0\nabla \times \nabla \times \mathbf{M} - i\omega\mu_0\nabla \times \mathbf{P}$$

question

How treat these equations?

negative refracting media

- first try

$$\left[\frac{\omega^2}{c^2} - \nabla \times \nabla\times\right] \mathbf{G}_e(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}') - \mu_0 \omega^2 \alpha_e \mathbf{G}_e(\mathbf{r}, \mathbf{r}') - i\omega\mu_0 \nabla \times \alpha_m \mathbf{G}_m(\mathbf{r}, \mathbf{r}')$$

$$\left[\frac{\omega^2}{c^2} - \nabla \times \nabla\times\right] \mathbf{G}_m(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}') - \mu_0 \nabla \times \nabla \times \alpha_m \mathbf{G}_m(\mathbf{r}, \mathbf{r}') + i\omega\mu_0 \nabla \times \alpha_e \mathbf{G}_e(\mathbf{r}, \mathbf{r}')$$

negative refracting media

- first try

$$\left[\frac{\omega^2}{c^2} - \nabla\times\nabla\times\right]\mathbf{G}_e(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}') - \mu_0\omega^2\alpha_e\mathbf{G}_e(\mathbf{r},\mathbf{r}') - i\omega\mu_0\nabla\times\alpha_m\mathbf{G}_m(\mathbf{r},\mathbf{r}')$$

$$\left[\frac{\omega^2}{c^2} - \nabla\times\nabla\times\right]\mathbf{G}_m(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}') - \mu_0\nabla\times\nabla\times\alpha_m\mathbf{G}_m(\mathbf{r},\mathbf{r}') + i\omega\mu_0\nabla\times\alpha_e\mathbf{G}_e(\mathbf{r},\mathbf{r}')$$

WRONG

problem

$\mathbf{G}_e(\mathbf{r},\mathbf{r}')$ and $\mathbf{G}_m(\mathbf{r},\mathbf{r}')$ have different source terms

negative refracting media

solution

- same source term for electric/magnetic Greens function

$$\mathbf{E}(\mathbf{r}) = -i\omega\mu_0 \int d\mathbf{r}' \tilde{\mathbf{G}}_e(\mathbf{r}, \mathbf{r}')\mathbf{j}(\mathbf{r}')$$

$$\mathbf{B}(\mathbf{r}) = -\mu_0 \int d\mathbf{r}' \tilde{\mathbf{G}}_m(\mathbf{r}, \mathbf{r}')\mathbf{j}(\mathbf{r}')$$

- drawback: different free Greens functions

$$\left[\frac{\omega^2}{c^2} - \nabla \times \nabla \times\right] \mathbf{G}_0^{(e)} = \delta \qquad \left[\frac{\omega^2}{c^2} - \nabla \times \nabla \times\right] \mathbf{G}_0^{(m)} = \nabla \times \delta$$

negative refracting media

resulting Dyson equations

$$\mathbf{G}_e(\mathbf{r},\mathbf{r}') = \mathbf{G}_0^{(e)}(\mathbf{r},\mathbf{r}') + \int d\mathbf{r}''\mathbf{G}_0^{(e)}(\mathbf{r},\mathbf{r}'')[-\mu_0\omega^2\alpha_e(\mathbf{r}'')\mathbf{G}_e(\mathbf{r}'',\mathbf{r}') - \mu_0\nabla''\times\alpha_m(\mathbf{r}'')\mathbf{G}_m(\mathbf{r}'',\mathbf{r}')]$$

$$\mathbf{G}_m(\mathbf{r},\mathbf{r}') = \mathbf{G}_0^{(m)}(\mathbf{r},\mathbf{r}') + \int d\mathbf{r}''\mathbf{G}_0^{(m)}(\mathbf{r},\mathbf{r}'')[-\mu_0\omega^2\alpha_e(\mathbf{r}'')\mathbf{G}_e(\mathbf{r}'',\mathbf{r}') - \mu_0\nabla''\times\alpha_m(\mathbf{r}'')\mathbf{G}_m(\mathbf{r}'',\mathbf{r}')]$$

negative refracting media

resulting Dyson equations

$$\mathbf{G}_e(\mathbf{r},\mathbf{r}') = \mathbf{G}_0^{(e)}(\mathbf{r},\mathbf{r}') + \int d\mathbf{r}''\mathbf{G}_0^{(e)}(\mathbf{r},\mathbf{r}'')[-\mu_0\omega^2\alpha_e(\mathbf{r}'')\mathbf{G}_e(\mathbf{r}'',\mathbf{r}') - \mu_0\nabla''\times\alpha_m(\mathbf{r}'')\mathbf{G}_m(\mathbf{r}'',\mathbf{r}')]$$

$$\mathbf{G}_m(\mathbf{r},\mathbf{r}') = \mathbf{G}_0^{(m)}(\mathbf{r},\mathbf{r}') + \int d\mathbf{r}''\mathbf{G}_0^{(m)}(\mathbf{r},\mathbf{r}'')[-\mu_0\omega^2\alpha_e(\mathbf{r}'')\mathbf{G}_e(\mathbf{r}'',\mathbf{r}') - \mu_0\nabla''\times\alpha_m(\mathbf{r}'')\mathbf{G}_m(\mathbf{r}'',\mathbf{r}')]$$

to be continued ...