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Slow light in ultra-cold atomic gases

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Abstract

We investigate the influence of slow light with an orbital angular momentum on the mechanical motion of ultra-cold atomic gases including both the atomic Bose–Einstein condensates and degenerate Fermi gases. We present a microscopic analysis of the interplay between light and matter and show how slow light can provide an effective magnetic field acting on the electrically neutral atoms.

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Bose–Einstein condensates (BECs) in atomic gases [1] have turned out to be a remarkable medium for studying a broad field of physics, ranging from fundamental atomic physics to cosmological aspects [2]. Recently, several experimental groups have succeeded in trapping and cooling fermions [3,4] well below the Fermi temperature. Fermi systems are well known from the study of electron properties in materials. On the other hand, BEC often acts like the real-life toy-model concept encountered in standard text books. A good example is the properties of BEC in optical lattices where atomic physics meets solid state physics.

Trapped atoms are electrically neutral, so a direct analogy between the magnetic properties of these systems and solid state phenomena is not necessarily straightforward. We suggest this problem can be circumvented if slow light is used, i.e., light with a group velocity as low as meters per second [5–7]. The coupling between the slow light and the atoms can give rise to some remarkable effects such as dragging of the light [8–10] and complete coherent freezing of the pulse [11–13]. In a similar manner, slow light should affect the atomic motion.

In this paper, we consider the influence of slow light on the mechanical properties of atomic BECs

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or degenerate Fermi gases of atoms. The theory is fully microscopic and based on the explicit analysis of the quantum dynamics of ultra-cold atoms coupled to the electromagnetic field. In particular, we use slow light with an orbital angular momentum [14,15]. This allows us to introduce an effective magnetic field which acts on the electrically neutral atoms. In the case of fermions we can have the typical, often regarded as an academic, textbook scenario with free electrons moving in a constant magnetic field. This opens up a possibility to study phenomena well known from solid state and condensed matter physics, with all the benefits given by the trapped atoms where a range of experimental parameters such as atom–atom interactions, particle numbers, the shape of the trapping potential, etc. can easily be manipulated.

Consider a system of atoms characterized by two hyper-fine ground levels 1 and 2, as well as an electronic excited level 3 (see Fig. 1). Initially the atoms occupy the lowest level 1. We shall describe the atoms in terms of the field operators $\Psi_j(\mathbf{r},t)$ representing the second-quantized wave function for the translational motion of atoms in the jth electronic state, with $j=1,2,3$. The operator $\Psi_j(\mathbf{r},t)$ annihilates an atom positioned at $\mathbf{r}$ and characterized by the internal state j. The operators $\Psi_j(\mathbf{r},t)$ can obey either Bose–Einstein or the Fermi–Dirac commutation relationships depending on the type of atoms involved. The atoms interact with two laser beams: A strong control laser drives the transition $|2\rangle \longrightarrow |3\rangle$, whereas a weaker probe field is coupled with the transition $|1\rangle \longrightarrow |3\rangle$ (see Fig. 1). In such an atomic medium, propagation of the probe field can be slowed down [5–7] by means of the electromagnetically induced transparency (EIT) [16–19], a phenomenon based on the quantum interference between the control and probe fields.

The control laser has a frequency ω_c, a wave vector $\mathbf{k}_c$, and a Rabi frequency $\Omega_c = \Omega_c^{(0)}\exp(\mathrm{i}\mathbf{k}_c\cdot\mathbf{r})$, where $\Omega_c^{(0)}$ is a slowly varying amplitude. The probe field, on the other hand, is characterized by a central frequency $\omega_p = ck_p$, a wave vector $\mathbf{k}_p = k_p\hat{\mathbf{z}}$, and a Rabi frequency $\Omega_p = \Omega_p^{(0)}\mathrm{e}^{\mathrm{i}(\ell\phi + k_p z)}$, where $\Omega_p^{(0)}$ is a slowly varying amplitude. Here, we have allowed the probe photons to have an

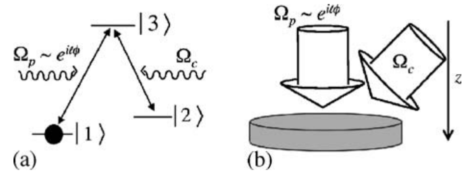

Fig. 1. (a) The level scheme for the electromagnetically induced transparency involving the probe beam Ω_p and control beam Ω_c. (b) Schematic representation of the experimental setup with the two light beams incident on the cloud of atoms. The probe beam propagates in the z-direction. The control beam can propagate parallel [12,13], perpendicular [5] or antiparallel to the probe beam.

orbital angular momentum $\hbar\ell$ along the z-axis [14,15].

Introducing the slowly-varying atomic field operators $\Phi_1 = \Psi_1\mathrm{e}^{\mathrm{i}\omega_1 t}$, $\Phi_3 = \Psi_3\mathrm{e}^{\mathrm{i}(\omega_1+\omega_p)t}$ and $\Phi_2 = \Psi_2\mathrm{e}^{\mathrm{i}(\omega_1+\omega_p-\omega_c)t}$, and adopting the rotating wave approximation, one can write the following equations of motion for the atomic field operators:

$$\mathrm{i}\hbar\dot{\Phi}_1 = -\frac{\hbar^2}{2m}\nabla^2\Phi_1 + V_1(\mathbf{r})\Phi_1 + \hbar\Omega_p^*\Phi_3, \tag{1}$$

$$\mathrm{i}\hbar\dot{\Phi}_3 = \left(\varepsilon_{31} - \frac{\hbar^2}{2m}\nabla^2\right)\Phi_3 + V_3(\mathbf{r})\Phi_3 + \hbar\Omega_c\Phi_2 + \hbar\Omega_p\Phi_1, \tag{2}$$

$$\mathrm{i}\hbar\dot{\Phi}_2 = \left(\varepsilon_{21} - \frac{\hbar^2}{2m}\nabla^2\right)\Phi_2 + V_2(\mathbf{r})\Phi_2 + \hbar\Omega_c^*\Phi_3, \tag{3}$$

where m is the atomic mass, $V_j(\mathbf{r})$ is the trapping potential for an atom in the electronic state j, $\varepsilon_{21} = \hbar(\omega_2 - \omega_1 + \omega_c - \omega_p)$ and $\varepsilon_{31} = \hbar(\omega_3 - \omega_1 - \omega_p)$ are, respectively, the energies of the detuning from the two- and single-photon resonances, $\hbar\omega_j$ being the electronic energy of the atomic level j.

Note that the equations of motion (1)–(3) do not accommodate collisions between the ground-state atoms. This is legitimate for the degenerate Fermi gas in which s-wave scattering is forbidden and only weak p-wave scattering is present [3,20–22]. In the case of an atomic BEC, the collisions can be included replacing Eq. (1) by the following mean-field equation for the condensate wave function Φ_1:

$$\mathrm{i}\hbar\dot{\Phi}_1 = -\frac{\hbar^2}{2m}\nabla^2\Phi_1 + V_1(\mathbf{r})\Phi_1 + g|\Phi_1|^2\Phi_1 + \hbar\Omega_p^*\Phi_3, \tag{4}$$

where $g = 4\pi\hbar^2 a/m$ and a is the s-wave scattering length. The scattering term in Eq. (4) will be disregarded in the subsequent discussion.

Suppose that the two-photon detuning ε_{21} is sufficiently small. Neglecting the terms with Φ_3, $\nabla^2\Phi_3$ and $\dot{\Phi}_3$ in Eq. (2), one arrives at the adiabatic condition [16–19] relating Φ_2 to Φ_1 as

$$\Phi_2(\mathbf{r}, t) = -\zeta\Phi_1(\mathbf{r}, t), \tag{5}$$

where $\zeta \equiv \Omega_p/\Omega_c$. Condition (5) holds if $\left|\left[i\hbar(\partial/\partial t)+(\hbar^2/2m)\nabla^2-\varepsilon_{31} - V_3(\mathbf{r})\right]\Phi_3\right| \ll \hbar|\Omega_p\Phi_1|$. This can be achieved if the spatial variation of the frequencies of two-photon recoil and two-photon Doppler shift is less than the Rabi frequency $|\Omega_c|$, as one can see from Eq. (6).

Condition (5) implies that the control and probe beams have driven the atoms to the dark state $|1\rangle - \zeta|2\rangle$ representing a special superposition between the two hyperfine ground states [16–19]. If the atoms are in the dark state, the resonant control and probe beams cannot populate the upper atomic level 3, as the two beams contribute destructively to the absorption process. This justifies neglecting the decay of the upper atomic level 3 in the equation of motion (2).

Eq. (5) shows that the orbital angular momentum $\hbar\ell$ of the probe field $\Omega_p \sim e^{i\ell\phi}$ is transferred into the orbital angular momentum of the centre of mass motion for atoms occupying level 2. This goes along with a general rule saying that the exchange of the orbital angular momentum in the electric dipole approximation occurs exclusively between the light and the atomic centre of mass motion [23]. The rule has been implicitly assumed in the initial equations of motion (1)–(3) containing no contributions due to exchange in the orbital angular momentum between the internal atomic states and the centre of mass motion.

Consider now the influence of the control and probe beams on the dynamics of the ground state atoms. Using Eqs. (3) and (5), one has

$$\Phi_3(\mathbf{r}, t) = -\frac{1}{\hbar\Omega_c^*}\left(\frac{\hbar^2}{2m}\nabla^2 + i\hbar\frac{\partial}{\partial t} - \varepsilon_{21} - V_2(\mathbf{r})\right) \times(\zeta\Phi_1). \tag{6}$$

Relationships (1) and (6) provide the following equation for the field operator Φ_1:

$$i\hbar\dot{\Phi}_1 = \frac{1}{2m}[i\hbar\nabla + \mathbf{A}_{\text{eff}}]^2\Phi_1 + V_{\text{eff}}(\mathbf{r})\Phi_1, \tag{7}$$

where

$$\mathbf{A}_{\text{eff}} = \frac{i\hbar\zeta^*\nabla\zeta}{1+|\zeta|^2} \equiv -\hbar\,\frac{|\zeta|^2}{1+|\zeta|^2}\,\nabla S + i\hbar\nabla\ln(1+|\zeta|^2)^{1/2} \tag{8}$$

and

$$V_{\text{eff}}(\mathbf{r}) = V_1(\mathbf{r}) + \frac{1}{2m}\frac{|\mathbf{A}_{\text{eff}}|^2}{|\zeta|^2} + \hbar\,\frac{(\omega_{21}|\zeta|^2 - i\zeta^*\frac{\partial}{\partial t}\zeta)}{1+|\zeta|^2} \tag{9}$$

are the *effective vector* and *trapping potentials*, and the dimensionless function $\zeta = e^{iS}\Omega_p^{(0)}/\Omega_c^{(0)}$ is characterized by a phase $S = (\mathbf{k}_p - \mathbf{k}_c)\cdot\mathbf{r} + \ell\phi$. Here $\hbar\omega_{21} = \varepsilon_{21} + V_2(\mathbf{r}) - V_1(\mathbf{r})$ is the modified energy of the two-photon detuning which includes the difference in trapping potentials. Note that the ratio $|\zeta|^2 \equiv |\Omega_p/\Omega_c|^2$ can be arbitrarily large in Eqs. (7)–(9), i.e., the intensity of the probe beam is not necessarily smaller than that of the control beam.

It is interesting to note that the *vector potential* $\mathbf{A}_{\text{eff}}$, given by Eq. (8), is generally non-Hermitian. This is because $\mathbf{A}_{\text{eff}}$ describes the dynamics of the atoms in level 1, from which some population is reversivibly transferred to level 2, as one can see from the adiabatic condition given by Eq. (5). The Hermitian contribution to $\mathbf{A}_{\text{eff}}$ is due to the changes in the phase S, the non-Hermitian one being due to the changes in the amplitude $|\zeta|$. The non-Hermitian part of $\mathbf{A}_{\text{eff}}$ can be eliminated by a pseudo-gauge transformation

$$\Phi_1 = \Phi_1^{(0)}\exp[-\ln(1+|\zeta|^2)^{1/2}] \equiv \Phi_1^{(0)}(1+|\zeta|^2)^{-1/2}. \tag{10}$$

In contrast to a previous paper by the authors [24], the transformation (10) is valid for arbitrary values of $|\zeta|^2$, i.e., the parameter $|\zeta|^2$ is not necessarily small. Note also, that both the probe and control fields (Ω_p and Ω_c) are considered to be incident quantities not affected by the induced motion of the ground-state fermions. If $|\zeta|^2 \ll 1$, the probe field Ω_p experiences a slow propagation at a group velocity $v_g \sim |\Omega_c|^2$ [16–19] in the z-direction.

In this way, we can create an effective vector potential through the phase S of the incoming probe beam. The experimental situation is schematically described in Fig. 1 where the incoming probe beam is of the form $e^{i\ell\phi}$.

With the vector potential we can define an effective magnetic field strength:

$$\mathbf{B}_{\text{eff}} = \nabla \times \mathbf{A}_{\text{eff}} = \hbar(\nabla S) \times \nabla \frac{|\zeta|^2}{1+|\zeta|^2}, \tag{11}$$

which is proportional to the orbital angular momentum of the probe beam. The presence of an effective magnetic field will have some important consequences. We are now in a position to study phenomena using ultra-cold neutral atomic gases, which have been previously considered only for electrons and charged bosons. One example is the de Haas-van Alphen effect. If we trap atomic fermions and apply the effective magnetic field the result will be an oscillation in the thermodynamical potentials as a function of the strength of the magnetic field [24]. Another example is an optical analog of the Meissner effect which could come about in atomic BEC by means of the effective magnetic field considered here.

In this paper, we have shown how light with an orbital angular momentum can be used to create an effective magnetic field in a degenerate gas of electrically neutral atoms (fermions or bosons). In particular, we derive the equations of motion for the case when the ratio between the probe and control beam is not necessarily small. There are a range of intriguing phenomena such as the quantum Hall effect, for instance, which can be studied using cold fermionic gases and slow light with an angular momentum. In addition, if the collisional interaction between the atoms is taken into account slow light can be used to study the magnetic properties of a superfluid atomic Fermi gas [25]. Recent advances in spatial light modulator technology enables us to consider rather exotic light beams [26]. This will allow us to study the effect of different forms of vector potentials in quantum gases. In particular, the combined dynamical system of light and matter could give important insight into gauge theories in general.

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