MODELING AND ESTIMATION OF $1/f$ NOISE OF THE SIGNALS REPRESENTED BY PULSES AND BY FLUCTUATING AMPLITUDE

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Generation and analysis of $1/f$ noise as consisting of pulses (point process) and represented by fluctuating amplitude of the signal are presented. It is shown how one type of the signal can be transformed into the other type with the same low frequency power spectral density.

1. Introduction

Long time correlations have been observed in different systems from physics to biology and sociology. The fact that $1/f$ noise is encountered in such a wide variety of systems has led to the speculations that there might exist some generic mechanism underlying the production of $1/f$ noise.

Recently [1, 2] we have shown that $1/f$ noise may be obtained in the point process approach with the Brownian motion of the interevent time, resulting in the clustering of the signal pulses [3, 4]. The purpose of this contribution is to relate such a process generating $1/f$ noise with more usual stochastic signals represented by fluctuating intensity (amplitude) of the signal.

2. Signals represented by pulses

The signal or intensity of a current of particles in some space cross section may be represented as consisting from pulses $A_k(t-t_k)$,

$$I(t)=\sum_k A_k(t-t_k), \tag{1}$$

with $\{t_k\}$ being a sequence of the pulses occurrence times t_k. It has been shown [1–4] that not for very long pulses $A_k(t-t_k)$ the intrinsic origin of $1/f$ noise may be the Brownian motion of the interevent time $\tau_k=t_k-t_{k-1}$, resulting sometimes in the clustering of the signal pulses.

The simplest version of such a signal is a point process, i.e., the signal represented by the Dirac $\delta(t-t_k)$ functions,

$$I(t)=a\sum_k \delta(t-t_k), \tag{2}$$

where a is the average area of the pulse.

The Brownian motion of the interevent time τ_k with some restrictions, e.g., with the relaxation to the average value $\bar{\tau}$ may be expressed as

$$\tau_k=|\tau_{k-1}-\gamma(\tau_{k-1}-\bar{\tau})+\sigma\varepsilon_k| \tag{3}$$

with $\{\varepsilon_k\}$ being a sequence of uncorrelated normally distributed random variables with zero expectation and unit variance and σ being the standard deviation of this white noise. The pulse occurrence times t_k are expressed as

$$t_k = t_{k-1} + \tau_k. \tag{4}$$

The power spectrum of signal generated by Eqs. (2)-(4) for small parameters σ and γ is $1/f$-like in any desirably wide range of frequencies [1, 2].

It should be noted that any signal $I(t)$ may be transformed into the point-like process by the integrate-and-fire method procedure for generating occurrence times t_k from the integrals [5]

$$\int_{t_{k-1}}^{t_k^{\pm}} I(t)dt = \pm a. \tag{5}$$

Then the signal $I(t)$ as a point process may be represented as

$$I(t) = a\sum_k \delta(t - t_k^+) - a\sum_k \delta(t - t_k^-). \tag{6}$$

Such a procedure and method have been used in the spectral analysis of the EKG signals for the predictions of a sudden cardiac death [6].

3. Signals represented by fluctuating intensity

We can introduce the rate of the signal as $\nu_k = 1/\tau_k$. Then from Eq. (3) we obtain the recurrent equation for the rate

$$\nu_k = \frac{\nu_{k-1}}{\left|1 - \gamma(1 - \bar{\nu}^{-1}\nu_{k-1}) + \sigma\nu_{k-1}\varepsilon_k\right|}. \tag{7}$$

Here $\bar{\nu} = 1/\bar{\tau}$ and the occurrence time t_k of the signal ν_k should be calculated as

$$t_k = \sum_{l=1}^{k} \tau_l = \sum_{l=1}^{k} \nu_l^{-1}. \tag{8}$$

Linearization of Eq. (7) yields

$$\nu_k = \left|\nu_{k-1} + \gamma\nu_{k-1}(1 - \bar{\nu}^{-1}\nu_{k-1}) + \sigma^2\nu_{k-1}^3 + \sigma\nu_{k-1}^2\varepsilon_k\right|. \tag{9}$$

From Eq. (9) we can derive the nonlinear Ito stochastic differential equation for $\nu(t)$ as function of the actual time t, i.e.,

$$\frac{d\nu}{dt} = \gamma\nu^2(1 - \bar{\nu}^{-1}\nu) + \sigma^2\nu^4 + \sigma\nu^{5/2}\xi(t). \tag{10}$$

Here $\xi(t)$ is δ-correlated, $\langle\xi(t)\xi(t')\rangle = \delta(t - t')$, white noise. The intensity of the signal is then $I(t) = a\nu(t)$.

Therefore, the appropriate nonlinear stochastic differential equation may generate the signal with the $1/f$ power spectral density, the same as the point process (2) with the fluctuating interevent time.

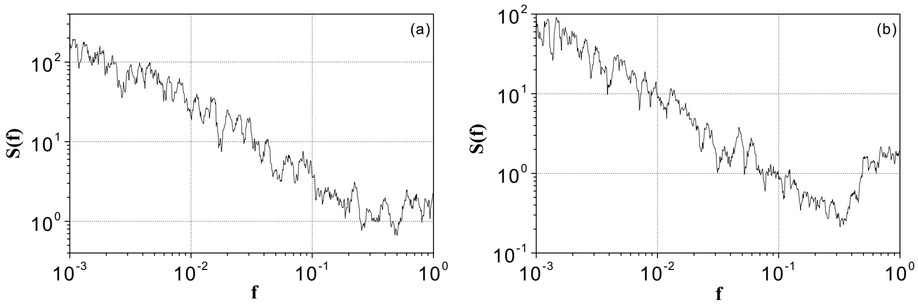

Fig. 1. Power spectra calculated a) for the point process (2)-(4) and b) from the recurrent equation for the signal (7).

Moreover, we can obtain the signal with the fluctuating amplitude from the point process (2) representing every pulse by the function $A_k(t-t_k)$ of the definite shape and duration, e.g., by the Gaussian overlapping pulses

$$I(t) = \frac{a}{\sqrt{2\pi}\sigma_p} \sum_k \exp\left\{-\frac{(t-t_k)^2}{2\sigma_p^2}\right\}. \tag{11}$$

Here σ_p^2 is the dispersion of the Gaussian pulse. Calculations of the power spectrum of the signal (11) by FFT yields at low frequencies the similar result as direct calculation of power spectrum of the point process (2) according to the expression

$$S_\delta(f) = \lim_{T\to\infty} \left\langle \frac{2a^2}{T} \left| \sum_k e^{-i2\pi f t_k} \right|^2 \right\rangle \tag{12}$$

with T being the observation time.

In general, the power spectral density of the signal (11) is

$$S(f) = S_p(f) S_\delta(f) \tag{13}$$

where $S_p(f) = exp\left\{-(2\pi f \sigma_p)^2\right\}$ is the spectrum of the individual Gaussian pulse. Therefore, we observe the cut of the spectrum of the signal (11) at $f \geq \sigma_p^{-1}$ and the spectrum without the shot noise. On the other hand, generation of the point-like process by the integrate-and-fire method (5)-(6) yields appearance of the shot noise at high frequencies. The low frequency noise in all cases is, however, the same.

Figure 1 represents the power spectral density of the signals generated by different procedures. We see the similarity of the behavior of the power spectra of the signals generated by different procedures and we demonstrate the possibility of generation of $1/f$ noise from the recurrent equations for the signal as well as for the interevent time.

4. Conclusions

The interrelation between the signals represented as consisting of pulses (point process) and more usual stochastic signals represented by fluctuating intensity is analyzed. It is shown how one type of the signal may be transformed into another type of the signal with the same power spectral density at low frequencies. The autoregressive equation for the interevent time τ_k of the point process may be transformed to the nonlinear Ito stochastic differential equation for the rate of the signal $\nu = 1/\tau_k$, resulting in the $1/f$ noise process. On the other hand, every signal may be transformed into the point-like process by the integrate-and-fire method for the generation of the occurrence times of the pulses. The low frequency noise of all signals obtained after such transformations is the same.

Acknowledgements

Support of the Lithuanian State Science and Studies Foundation is acknowledged.

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