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Creation of an Effective Magnetic Field in Ultracold Atomic Gases Using Electromagnetically Induced Transparency¹

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Abstract—We consider the influence of the control and probe beams in the electromagnetically induced transparency configuration on the mechanical motion of ultracold atomic gases (atomic Bose–Einstein condensates or degenerate Fermi gases). We carry out a microscopic analysis of the interplay between radiation and matter and show that the two beams of light can provide an effective magnetic field acting on electrically neutral atoms in the case where the probe beam has an orbital angular momentum. As an example, we demonstrate how a Meissner-like effect can be created in an atomic Bose–Einstein condensate. © 2005 Pleiades Publishing, Inc.

1. INTRODUCTION

Atomic Bose–Einstein condensates (BECs) [1] have proven to be a remarkable medium for studying phenomena in areas ranging from fundamental atomic physics to cosmological aspects [2]. Recently, several experimental groups have also succeeded in trapping and cooling atomic fermions [3, 4] well below the Fermi temperature. Fermi systems are well known from the study of electron properties in materials. On the other hand, a BEC often acts like the real-life model concept encountered in standard textbooks. A good example is the BEC in optical lattices, where atomic physics meets solid state physics.

The concept of a BEC as a superfluid makes it tempting to try to find analogies between these quantum gases and the properties of superconductors. One example is the Meissner effect [5] in superconductors, where the magnetic field penetrates the sample and induces vortices. At first glance, it is certainly not clear how to obtain a similar situation in a BEC since the Meissner effect relies on the properties of the vector potential that describes the magnetic field.

A BEC consists of neutral atoms and is therefore not affected by the magnetic field as in a superconductor. It is, however, possible to have a vector potential–like term in a BEC or a degenerate Fermi gas if the atomic gases interact with control and probe beams of light in an electromagnetically induced transparency (EIT) configuration [6]. Application of the control beam is known to lead to a dramatic reduction of the group velocity of the probe beam, which can be as low as meters per second [7–9]. The coupling between the slow light and the atoms can give rise to some remarkable effects, such as dragging of the light [10–12] and complete coherent freezing of the pulse [13–15]. In a

similar manner, the control and probe beams should affect the atomic motion.

In this paper, we consider the influence of the control and probe beams of light on the mechanical properties of atomic BECs or degenerate Fermi gases of atoms. The theory is fully microscopic and is based on the explicit analysis of the quantum dynamics of ultracold atoms coupled to two beams of light. We show that the application of a probe beam with an orbital angular momentum [16, 17] leads to an effective magnetic field that acts on the electrically neutral atoms. This opens up the possibility of studying magnetic phenomena well known from solid state and condensed matter physics with all the benefits given by trapped atoms, where a range of experimental parameters such as atom–atom interactions, particle numbers, the shape of the trapping potential, etc., can easily be manipulated. As an example, we show how an optical analogue of the Meissner effect comes about in atomic BECs.

2. FORMULATION

Consider an ensemble of cold atoms characterized by two hyperfine ground levels 1 and 2, as well as an electronic excited level 3 (Fig. 1). Initially, the atoms occupy the lowest level 1. We shall describe the atoms in terms of the field operators $\Psi_j(\mathbf{r}, t)$ representing the second-quantized wave function for the translational motion of atoms in the j th electronic state, with $j = 1, 2, 3$. The operator $\Psi_j(\mathbf{r}, t)$ annihilates an atom positioned at \mathbf{r} and is characterized by the internal state j . The operators $\Psi_j(\mathbf{r}, t)$ can obey either Bose–Einstein or Fermi–Dirac commutation relationships depending on the type of atoms involved. The atoms interact with two laser beams: A control laser drives the transition $|2\rangle \rightarrow |3\rangle$, whereas a probe field is coupled with the transition $|1\rangle \rightarrow |3\rangle$ (Fig. 1). In such a situation, the propagation

¹ The text was submitted by the authors in English.

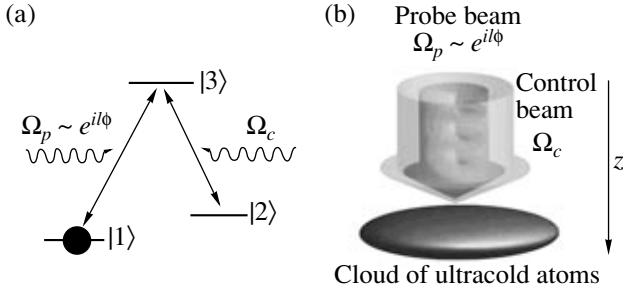


Fig. 1. (a) The level scheme for the EIT involving the probe beam Ω_p and control beam Ω_c . (b) Schematic representation of the experimental setup with the two light beams incident on the cloud of atoms. The probe field is of the form $\Omega_p \sim e^{i l \phi}$, so the probe photons are allowed to have an orbital angular momentum $\hbar l$ along the propagation axis z .

of a weak probe field can be slowed down [7–9] by means of EIT [18–21], a phenomenon based on quantum interference between the control and the probe fields.

The control laser has a frequency ω_c , a wave vector \mathbf{k}_c , and a Rabi frequency

$$\Omega_c = \Omega_c^{(0)} e^{i \mathbf{k}_c \cdot \mathbf{r}}, \quad (1)$$

where $\Omega_c^{(0)}$ is a slowly varying amplitude. The probe field, on the other hand, is characterized by a central frequency $\omega_p = ck_p$, a wave vector $\mathbf{k}_p = k_p \hat{z}$, and a Rabi frequency

$$\Omega_p = \Omega_p^{(0)} e^{i(l\phi + \mathbf{k}_p \cdot \mathbf{r})}, \quad (2)$$

where $\Omega_p^{(0)}$ is a slowly varying amplitude and ϕ is the azimuthal angle. In writing Eq. (2), we have allowed the probe photons to have an orbital angular momentum $\hbar l$ along the propagation axis z [16, 17].

Let us introduce the slowly varying atomic field operators $\Phi_1 = \Psi_1 e^{i\omega_1 t}$, $\Phi_3 = \Psi_3 e^{i(\omega_1 + \omega_p)t}$, and $\Phi_2 = \Psi_2 e^{i(\omega_1 + \omega_p - \omega_c)t}$. Adopting the rotating wave approximation, one can write the following equations of motion for the atomic field operators:

$$i\hbar \dot{\Phi}_1 = -\frac{\hbar^2}{2m} \nabla^2 \Phi_1 + V_1(\mathbf{r}) \Phi_1 + \hbar \Omega_p^* \Phi_3, \quad (3)$$

$$i\hbar \dot{\Phi}_3 = \left(\epsilon_{31} - \frac{\hbar^2}{2m} \nabla^2 \right) \Phi_3 + V_3(\mathbf{r}) \Phi_3 + \hbar \Omega_c \Phi_2 + \hbar \Omega_p \Phi_1, \quad (4)$$

$$i\hbar \dot{\Phi}_2 = \left(\epsilon_{21} - \frac{\hbar^2}{2m} \nabla^2 \right) \Phi_2 + V_2(\mathbf{r}) \Phi_2 + \hbar \Omega_c^* \Phi_3, \quad (5)$$

where m is the atomic mass; $V_j(\mathbf{r})$ is the trapping potential for an atom in the electronic state j ; and $\epsilon_{21} = \hbar(\omega_2 - \omega_1 + \omega_c - \omega_p)$ and $\epsilon_{31} = \hbar(\omega_3 - \omega_1 - \omega_p)$ are, respectively,

the energies of the detuning from the two- and single-photon resonances, $\hbar\omega_j$ being the electronic energy of the atomic level j .

Equations of motion (3)–(5) do not accommodate collisions between the ground state atoms. This is legitimate for the degenerate Fermi gas, in which s -wave scattering is forbidden and only weak p -wave scattering is present [3, 22–24]. If the atoms in ground electronic state 1 form a BEC, the atomic collisions can be included replacing Eq. (3) by the following mean field equation of the condensate wave function Φ_1 :

$$i\hbar \dot{\Phi}_1 = -\frac{\hbar^2}{2m} \nabla^2 \Phi_1 + V_1(\mathbf{r}) \Phi_1 + g_1 |\Phi_1|^2 \Phi_1 + \hbar \Omega_p^* \Phi_3, \quad (6)$$

where $g_1 = 4\pi\hbar^2 a_1/m$ and a_1 is the s -wave scattering length for the atoms in electronic state 1.

3. ADIABATIC APPROXIMATION

Suppose that the two-photon detuning ϵ_{21} is sufficiently small. Neglecting the terms with Φ_3 , $\nabla^2 \Phi_3$, and $\dot{\Phi}_3$ in Eq. (4), one arrives at the adiabatic condition [18–21] that relates Φ_2 to Φ_1 :

$$\Phi_2(\mathbf{r}, t) = -\zeta \Phi_1(\mathbf{r}, t), \quad (7)$$

where $\zeta \equiv \Omega_p/\Omega_c$. Condition (7) holds if $\left| \left[i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2}{2m} \nabla^2 - \epsilon_{31} - V_3(\mathbf{r}) \right] \Phi_3 \right| \ll \hbar |\Omega_p \Phi_1|$. This can be

achieved if the spatial variation of the frequencies of the two-photon recoil and the two-photon Doppler shift is less than the Rabi frequency $|\Omega_c|$, as one can see from subsequent equation (8).

Condition (7) implies that the control and probe beams have driven the atoms to the dark state $|1\rangle - \zeta|2\rangle$, representing a special superposition of the two hyperfine ground states [18–21]. If the atoms are in the dark state, the resonant control and probe beams cannot populate upper atomic level 3 since the two beams contribute destructively to the absorption process due to quantum interference [18–21].

Equation (7) shows that the orbital angular momentum $\hbar l$ of the probe field $\Omega_p \sim e^{i l \phi}$ is transferred to the orbital angular momentum of the center of mass motion for atoms occupying electronic level 2. This goes along with a general rule saying that the exchange of the orbital angular momentum in the electric dipole approximation occurs exclusively between the light and the atomic center of mass motion [25]. The rule has been implicitly assumed in the initial equations of motion (3)–(5) and (6). These equations contain no contributions due to exchange of the orbital angular momentum between the internal atomic states and the center of mass motion.

4. EFFECTIVE EQUATION OF MOTION FOR Φ_1

Consider now the influence of the control and probe beams on the dynamics of the ground state atoms. Using Eqs. (5) and (7), one has

$$\begin{aligned} & \Phi_3(\mathbf{r}, t) \\ &= -\frac{1}{\hbar\Omega_c^*} \left(\frac{\hbar^2}{2m} \nabla^2 + i\hbar \frac{\partial}{\partial t} - \epsilon_{21} - V_2(\mathbf{r}) \right) (\zeta \Phi_1). \end{aligned} \quad (8)$$

Relationships (3) and (8) provide the following closed equation for the field operator Φ_1 :

$$i\hbar \dot{\Phi}_1 = \frac{1}{2m} [i\hbar \nabla + \mathbf{A}_{\text{eff}}]^2 \Phi_1 + V_{\text{eff}}(\mathbf{r}) \Phi_1, \quad (9)$$

where

$$\begin{aligned} \mathbf{A}_{\text{eff}} &= \frac{i\hbar \zeta^* \nabla \zeta}{1 + |\zeta|^2} \\ &\equiv -\hbar \frac{|\zeta|^2}{1 + |\zeta|^2} \nabla S + i\hbar \nabla \ln(1 + |\zeta|^2)^{1/2} \end{aligned} \quad (10)$$

and

$$V_{\text{eff}}(\mathbf{r}) = V_1(\mathbf{r}) + \frac{1}{2m} \frac{|\mathbf{A}_{\text{eff}}|^2}{|\zeta|^2} + \hbar \frac{\left(\omega_{21} |\zeta|^2 - \zeta^* \frac{\partial}{\partial t} \zeta \right)}{1 + |\zeta|^2} \quad (11)$$

are the effective vector and trapping potentials and the dimensionless function $\zeta = e^{iS} \Omega_p^{(0)} / \Omega_c^{(0)}$ is characterized by a phase $S = (\mathbf{k}_p - \mathbf{k}_c) \mathbf{r} + l\phi$. Here $\hbar\omega_{21} = \epsilon_{21} + V_2(\mathbf{r}) - V_1(\mathbf{r})$ is the modified energy of the two-photon detuning, which includes the difference in trapping potentials. In contrast to our previous paper [6], the ratio $|\zeta|^2 \equiv |\Omega_p / \Omega_c|^2$ can be arbitrarily large in Eqs. (9)–(11). In other words, the intensity of the probe beam is not necessarily smaller than that of the control beam.

It is instructive to note that the vector potential \mathbf{A}_{eff} describing an effective dynamics of atoms in electronic state 1 is generally non-Hermitian. This is because the probe and control beams reversibly transfer some atomic population from level 1 to level 2 by means of the two-photon Raman transition, as one can see from the adiabatic condition given by Eq. (7). Therefore, the non-Hermitian \mathbf{A}_{eff} is an operator of an open subsystem. The Hermitian contribution to \mathbf{A}_{eff} is due to the changes in the phase S , the non-Hermitian one being due to the changes in the amplitude $|\zeta| \equiv |\Omega_p / \Omega_c|$. The non-Hermitian part of \mathbf{A}_{eff} can be eliminated by a pseudogauge transformation

$$\begin{aligned} \Phi_1 &= \Phi_1^{(0)} \exp[-\ln(1 + |\zeta|^2)^{1/2}] \\ &\equiv \Phi_1^{(0)} (1 + |\zeta|^2)^{-1/2}. \end{aligned} \quad (12)$$

It is noteworthy that transformation (12) is valid for arbitrary values of $|\zeta|^2$; i.e., the parameter $|\zeta|^2$ is not necessarily small. Note also that both the probe and the

control fields (Ω_p and Ω_c) are considered to be incident quantities not affected by the induced motion of the ground state fermions. If $|\zeta|^2 \ll 1$, the probe field Ω_p experiences slow propagation at a group velocity $v_g \sim |\Omega_c|^2$ [18–21] in the z direction.

In the case of a BEC, the collisions between the ground state atoms can be included replacing Eq. (3) by Eq. (6). In such a situation, effective equation of motion (9) is modified,

$$\begin{aligned} & i\hbar \dot{\Phi}_1 \\ &= \frac{1}{2m} [i\hbar \nabla + \mathbf{A}_{\text{eff}}]^2 \Phi_1 + V_{\text{eff}}(\mathbf{r}) \Phi_1 + g |\Phi_1|^2 \Phi_1, \end{aligned} \quad (13)$$

with $g = g_1 / (1 + |\zeta|^2)$, where \mathbf{A}_{eff} and V_{eff} are the same as in Eqs. (10) and (11).

The experimental situation is schematically described in Fig. 1, where the incoming probe beam is of the form $\Omega_p \sim e^{i\phi}$. In such a situation, we can create an effective vector potential through the phase $S \equiv l\phi$ of the incoming probe beam. With vector potential (10), we can obtain an effective magnetic field strength

$$\mathbf{B}_{\text{eff}} = \nabla \times \mathbf{A}_{\text{eff}} = \hbar l (\nabla \phi) \times \nabla \frac{|\zeta|^2}{1 + |\zeta|^2}, \quad (14)$$

which is proportional to the orbital angular momentum $\hbar l$. The presence of an effective magnetic field will have some important consequences. We are now in a position to study phenomena using ultracold neutral atomic gases, which have previously only been considered for electrons and charged bosons. One example is the de Haas–van Alphen effect, considered previously [6]. If we trap atomic fermions and apply an effective magnetic field, the thermodynamic potentials will oscillate as a function of the magnetic field strength [6]. Another example is an optical analogue of the Meissner effect that could come about in atomic BECs by means of the effective magnetic field, as we shall see next.

5. MEISSNER-LIKE EFFECTS IN AN ATOMIC BEC

Consider a condensate trapped in a cylindrical container with radius R . Such an external trap can be created by, for instance, high-order Bessel beams [26, 27]. Suppose that $|\zeta|^2 = |\Omega_p|^2 / |\Omega_c|^2 \ll 1$ and that the intensity of the control beam $|\Omega_c|^2$ does not vary considerably within the atomic cloud. If the control and probe beams are copropagating and we choose the intensity of the probe beam of the form $|\Omega_p|^2 \sim r^2$ in the transversal plane, we obtain the following effective vector potential:

$$\mathbf{A}_{\text{eff}} = \frac{\hbar l \alpha_0}{R^2} (-y \hat{\mathbf{e}}_x + x \hat{\mathbf{e}}_y), \quad (15)$$

where $\alpha_0 = |\zeta|^2 R^2 / r^2$ is the small ratio (typically less than 0.1) between the probe beam and the control beam

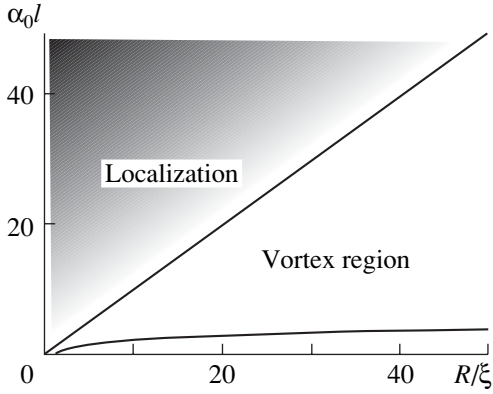


Fig. 2. The phase diagram for vortices in the condensate as a function of the size R and the strength of the effective vector potential $\alpha_0 l$. The two lines correspond to the two limits in Eq. (22).

at radius R of the cylinder in which the gas is contained. It is interesting to note here that, with this choice of light, effective vector potential (15) corresponds to a constant magnetic field in the z direction:

$$\mathbf{B}_{\text{eff}} = \nabla \times \mathbf{A} = \frac{2\hbar l \alpha_0}{R^2} \hat{\mathbf{e}}_z. \quad (16)$$

The strength of the effective magnetic field is given by the orbital angular momentum of the probe photons $\hbar l$ and can be controlled by applying suitable phase and intensity holograms [28]. It is relatively straightforward to create and control high angular momenta of the order of $l \leq 1000$ that consequently control the effective magnetic field.

With the effective vector potential given by Eq. (15), we obtain the expression for the total energy,

$$E[\Psi] = \int d\mathbf{r} \left[\frac{1}{2m} |(i\hbar \nabla + \mathbf{A}_{\text{eff}})\Psi|^2 + \frac{1}{2} g |\Psi|^4 \right] \quad (17)$$

$$= \int d\mathbf{r} \left[\frac{\hbar^2}{2m} |\partial_r \Psi|^2 + \frac{\hbar^2}{2m} \left(\beta r - \frac{\nu}{r} \right)^2 |\Psi|^2 + \frac{1}{2} g |\Psi|^4 \right], \quad (18)$$

with $\beta = \alpha_0 l / R^2$, where we have allowed for the possibility of a vortex solution, $\Psi \equiv \Phi_1 \sim e^{i\nu\phi}$. Here, we have in addition chosen the external potential such that $V_{\text{eff}}(r) = 0$. From Eq. (18), we see that the effective vector potential induces a harmonic potential with the cyclotron frequency

$$\omega = \frac{\hbar}{m} \alpha_0 l \frac{1}{R^2}, \quad (19)$$

which acts like a localization potential. The higher the orbital angular momentum $\hbar l$, the stronger the induced

cyclotron frequency. If we assume that $R \ll \frac{2\mu}{m\omega^2}$,

where μ is the chemical potential of the condensate, we see that the total energy of the condensate with winding number ν is of the form

$$E_\nu = E_{\nu=0} + \frac{\hbar^2}{2m} \int_{\xi}^R dr r \left(\frac{\nu^2}{r^2} - 2\beta \right) |\Psi(r)|^2 \quad (20)$$

$$= E_{\nu=0} + \frac{\hbar^2}{2m} \left(\nu^2 \ln \left(\frac{R}{\xi} \right) - \beta \nu R^2 \right), \quad (21)$$

where $\xi = \sqrt{\frac{\hbar}{mg\rho_0}}$ is the condensate healing length, ρ_0

is the atomic density in the center of the trap with no vortex present, and γ is a numerical constant of the order of unity [29]. If we neglect the size of the vortex core, we obtain the critical angular momentum of the light,

$$\nu \ln \left[\frac{R}{\xi} \right] < \alpha_0 l \ll \frac{R}{\xi}, \quad (22)$$

where the right-hand side comes from the fact that we consider weak localization. In Fig. 2, we show the phase diagram for vortices as a function of R and $\alpha_0 l$. The localization effect studied in the previous section will eventually distort the phase front of the light due to the change in the density of the condensate. One way to avoid this is to choose a different effective vector potential of the form where the phase is still the same as previously but the intensity of the light is constant,

$$\mathbf{A}_{\text{eff}} = \hbar \alpha_0 \frac{l}{r} \hat{\mathbf{e}}_\phi. \quad (23)$$

The corresponding total energy is then of the form

$$E[\Psi] = \int d\mathbf{r} \left[\frac{\hbar^2}{2m} |\partial_r \Psi|^2 + \frac{\hbar^2}{2mr^2} (\alpha_0 l - \nu)^2 |\Psi|^2 + \frac{1}{2} g |\Psi|^4 \right], \quad (24)$$

where the induced potential now is of a centrifugal form. From energy functional (24), it is clear that the energy will be minimized whenever

$$\alpha_0 l = \nu = 1, 2, 3, \dots \quad (25)$$

In other words, vortex solutions will be favored whenever the angular momentum times the quantity α_0 is an integer value. It is, however, important to remember that for $\nu > 1$ the vortex is not stable and will break up into vortices with unit winding numbers [30].

The creation of vortices by using a vector potential very much resembles the Meissner effect in type II superconductors. In a condensate, however, the concept of a penetration depth is not relevant since in our case the light by definition propagates through the conden-

sate. The analogy lies in the fact that vortices are created at a critical value for the effective magnetic field. This, admittedly, is of course also the same as inducing vorticity in the system.

It is interesting to note that the technique presented here can also be used where the condensate is trapped in a toroidal external trap. A torus trap can be created using Laguerre–Gauss beams [27] or magnetic traps [31]. The vortex state would in this case correspond to a persistent current with the phase $\Psi \sim e^{ip\phi}$, where p is an integer. The situation can effectively be reduced to a one-dimensional problem if the confinement is sufficiently strong, $\mu \leq \hbar\omega_r$, where ω_r is the radial trapping frequency in the torus.

6. CONCLUSIONS

In this paper, we have shown how the probe beam of light with an orbital angular momentum can produce an effective magnetic field in a degenerate gas of electrically neutral atoms (fermions or bosons) using EIT. We have derived an effective equation of atomic motion containing vector potential–type interaction in the case where the ratio between the intensities of the probe and control beams is not necessarily small. We have demonstrated that the effective vector potential can lead to an optical analogue of the Meissner effect in an atomic BEC. Our theory can be applied to other intriguing phenomena that intrinsically depend on the magnetic field. For instance, the quantum Hall effect can now be studied using a cold gas of electrically neutral atomic fermions. In addition, if the collisional interaction between the atoms is taken into account, we can study the magnetic properties of a superfluid atomic Fermi gas [32]. Recent advances in spatial light modulator technology enable us to consider rather exotic light beams [33]. This will allow us to study the effect of different forms of vector potentials in quantum gases. In particular, the combined dynamical system of light and matter [34] could give an important insight into gauge theories in general.

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