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Absorption by cold Fermi atoms in a harmonic trap

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We study the absorption line shape of a strongly degenerate Fermi gas confined in a harmonic trap. The absorption spectrum is calculated using both the exact summation and also the Thomas-Fermi approximation. In the latter case, relatively simple analytical expressions are obtained for the absorption line shape for a large number of trapped atoms. At zero temperature, the approximated line shape is characterized by a $(1-z^2)^{5/2}$ dependence, which agrees well with the exact numerical calculations. At nonzero temperature, the spectrum becomes broader, although remains non-Gaussian as long as the fermion gas is degenerate. The changes in the trap frequency for an electronically excited atom can introduce an additional line broadening.

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In recent years there has been a great deal of interest in the dilute gas of trapped atoms cooled to temperatures below 1 μK. At such low temperatures, an important role is played by the quantum statistics of atoms. Bosons tend to occupy the lowest translational level of the trap to form the Bose-Einstein condensate [1]. On the other hand, fermions can still occupy many trap levels (predominantly up to the Fermi level), as was observed recently on ^{40}K atoms [2]. Trapping and cooling of ^{6}Li [3] and ^{171}Yb [4] fermion atoms are also under way.

Optical spectroscopy has proven to be useful in getting information about cold atomic gases [1,2,5–10]. The Fermi-Dirac (FD) statistics is known to change the optical response of the system compared to the classical one. The signatures of quantum degeneracy emerge in the scattering spectra of homogeneous [6] and trapped [8–10] Fermi gases. Furthermore, the spontaneous emission appears to be inhibited in a cold Fermi gas [7].

The effects of quantum degeneracy should manifest in the absorption spectra as well. The aim of the present paper is to investigate absorption by a cold Fermi gas confined in a harmonic trap. The analysis concentrates on the degenerate Fermi gas (i.e., very low temperatures), for which the quantum statistics of the atoms plays an important role. The theory involves exact calculations, as well as the Thomas-Fermi (TF) approximation. Consequently, the absorption spectrum is analyzed both for small and large numbers of trapped atoms.

Consider a gas of Fermi atoms confined in a harmonic trap. The atoms are assumed to be in the identical magnetic sublevels of the hyperfine ground state, as is the case in the experiment [2]. In such a situation, the *s*-wave collisions are forbidden [2,3,11–13] and the *p*-wave collisions are suppressed at low temperature due the centrifugal barrier [8,14,15]. Consequently one can make use of the following one-atom Hamiltonian:

$$H_{1-at}=|g\rangle H_g\langle g|+\sum_{\rm ex}|{\rm ex}\rangle(\hbar\omega_{0,{\rm ex}}+H_{\rm ex})\langle{\rm ex}|, \qquad (1)$$

with

$$H_{g,{\rm ex}}(\mathbf{r},\mathbf{p})=\frac{p^2}{2M}+\frac{M\Omega_{g,{\rm ex}}^2(x^2+\lambda_y^2y^2+\lambda_z^2z^2)}{2}, \qquad (2)$$

where $\hbar\omega_{0,{\rm ex}}$ is the electronic excitation energy, H_g ($H_{\rm ex}$) is the translational Hamiltonian for a trapped atom in the ground $|g\rangle$ (excited $|{\rm ex}\rangle$) electronic state, $\Omega_{g,{\rm ex}}$ is the frequency of the translational motion along the x axis, $\mathbf{p}=-i\hbar\nabla$ is the momentum operator, M is the atomic mass, and the dimensionless parameters λ_y and λ_z describe the extent of anisotropy of the trap. Note that the ground-state frequency Ω_g can be generally different from $\Omega_{\rm ex}$ due to the changes in the magnetic moment of the atom following its transition to an excited electronic state. The effects related to this fact will be explored in terms of the TF approximation.

The line shape of the absorption spectrum is given by

$$I(\omega)=\sum_{i,f}\rho_i|\langle f|V|i\rangle|^2\delta(\omega-\omega_{fi}). \qquad (3)$$

Here $|i\rangle\equiv|g\rangle|\mathbf{n}\rangle_g$ and $|f\rangle\equiv|{\rm ex}\rangle|\mathbf{m}\rangle_{\rm ex}$ are the initial and final states of an atom, $|\mathbf{n}\rangle_g\equiv|n_x,n_y,n_z\rangle_g$ and $|\mathbf{m}\rangle_{\rm ex}\equiv|m_x,m_y,m_z\rangle_{\rm ex}$ are the atomic translational states characterized by the energies

$$\varepsilon_{\mathbf{n}}^{g,{\rm ex}}=\hbar\Omega_{g,{\rm ex}}(n_x+\lambda_yn_y+\lambda_zn_z)+\varepsilon_{\mathbf{0}}^{g,{\rm ex}}, \qquad (4)$$

where $\varepsilon_{\mathbf{0}}^{g,{\rm ex}}=\hbar\Omega_{g,{\rm ex}}(1+\lambda_y+\lambda_z)/2$, and $\omega_{fi}=\omega_{0,{\rm ex}}+(\varepsilon_{\mathbf{m}}^{\rm ex}-\varepsilon_{\mathbf{n}}^{g})/\hbar$ is the transition frequency. Here also $\rho_i\equiv\rho(\varepsilon_{\mathbf{n}}^{g})=[\exp(\beta\varepsilon_{\mathbf{n}}^{g}-\beta\mu)+1]^{-1}$ is the FD distribution function for the trapped atoms, μ is the chemical potential, and $\beta=1/k_BT$. The operator

$$V=\sum_{\rm ex}\{|{\rm ex}\rangle d_{ex}\exp(i\kappa x)\langle g|+{\rm H.c.}\} \qquad (5)$$

describes the interaction between an atom and the electromagnetic field propagating along the x axis, κ is the wave number of the light, and $d_{\rm ex}$ is the atomic transition dipole moment along the polarization of the light.

Consider first the absorption line shape exactly. At this stage, it is assumed that $\Omega_g=\Omega_{\rm ex}=\Omega$, yet the trap can still

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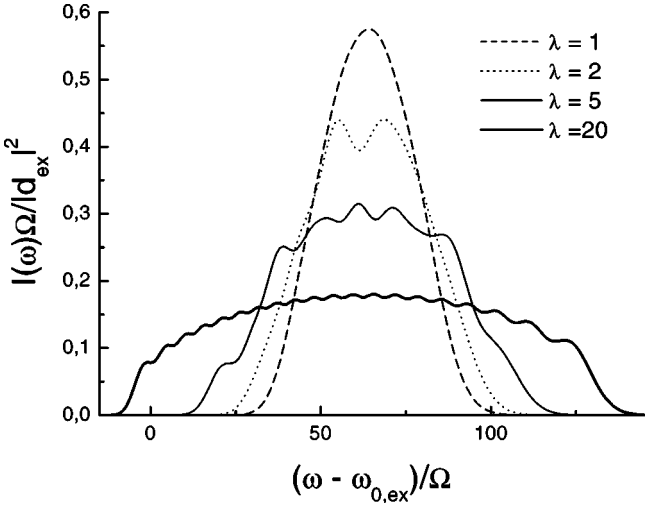

FIG. 1. Absorption line shape calculated using Eq. (6) for $N=20$, $T=0$, and $\alpha=8$. The singular contributions due to the δ functions have been smoothed.

be anisotropic. At zero temperature [17], only the levels with $n_x+\lambda_y n_y+\lambda_z n_z \leqslant n_F=E_F/\hbar\Omega$ are occupied by the atoms, where $E_F \equiv \mu|_{T=0}$ is the Fermi energy. Exploiting the matrix elements $\langle n_x'|\exp(i\kappa x)|n_x\rangle$ of the harmonic oscillator [16], the line shape (3) takes the form

$$I(\omega)=\sum_{\mathrm{ex}}|d_{\mathrm{ex}}|^2\sum_{n_x=0}^{n_F}\sum_{m=-n_x}^{\infty}K_x\delta(\omega-\omega_{0,\mathrm{ex}}-m\Omega)\times n_x!(n_x+m)!e^{-\alpha^2}(\alpha^2)^{2n_x+m}\times\left(\sum_{j=0}^{j_x}\frac{(-\alpha^{-2})^j}{j!(n_x-j)!(n_x+m-j)!}\right)^2, \tag{6}$$

with $j_x=\min(n_x,n_x+m)$, where $\alpha=\kappa(\hbar/2M\Omega)^{1/2}$, and the factor $K_x=\Sigma_{n_y=0}^{[(1/\lambda_y)(n_F-n_x)]}\Sigma_{n_z=0}^{[(1/\lambda_z)(n_F-n_x-\lambda_y n_y)]}1$ represents a number of occupied translational states $|n_x,n_y,n_z\rangle$ for a fixed value of n_x, the brackets [ ] labeling the integer part of a number. The Fermi number n_F is determined by the condition $\Sigma_{n_x=0}^{n_F}K_x=N$, where N is the number of trapped atoms. The factor K_x reflects the trap geometry. For traps with a cylindric symmetry ($\lambda_y=\lambda_z=\lambda$), one finds $K_x=([q_x]+1)([q_x]+2)/2$, where $q_x=(n_F-n_x)/\lambda$. For an anisotropic trap of a cigar shape ($\lambda_y,\lambda_z \gg 1$), one has $K_x=1$ provided the number of trapped atoms is small enough ($n_F<\lambda_y,\lambda_z$). In such a situation, the trap becomes one-dimensional (1D), giving $n_F=N-1$. On the other hand, for an isotropic three-dimensional (3D) trap ($\lambda_y=\lambda_z=1$), one arrives at $K_x=(n_F-n_x+1)(n_F-n_x+2)/2$. If the number of atoms is sufficiently large ($n_F>\lambda_y,\lambda_z$), the anisotropic traps of cigar shape ($\lambda_y,\lambda_z \gg 1$) are no longer one dimensional, since the Fermi energy is then greater than the energy of the translational quanta in the y and z directions. Such a situation corresponds to the recent experiment by DeMarco and Jin [2].

Figure 1 shows the absorption line shapes for $\lambda_y=\lambda_z=\lambda$. A single excited electronic state $|\mathrm{ex}\rangle$ has been taken into account in these and the subsequent figures. Oscillations are featured in the thick solid line representing a purely 1D case ($\lambda=20$). This reflects the Friedel oscillations in the atomic number density [18] due to the finite size of the system. The oscillations are also seen in the line shape of an anisotropic 3D trap of the cigar shape ($\lambda=5$). In such a case, the energy of the translational quanta perpendicular to the trap axis is much larger than that along the x axis, so a one-dimensional character of the spectrum might be preserved. Oscillations do not appear in the line shape of an isotropic 3D trap ($\lambda=1$).

Note also that in contrast to a single trapped atom [19], the zero-temperature line shape of the trapped Fermi gas has a cutoff at a frequency smaller than $\omega_{0,\mathrm{ex}}$. This can be explained by the Pauli exclusion principle enabling the fermions to occupy excited translational levels of the trap at $T=0$ (up to the Fermi level). Hence the optical absorption can be accompanied by a decrease in the translational energy of the atoms. For the same reason the absorption spectrum becomes broader as the number of trapped fermions increases. This is in contrast to an ideal classical gas, the Doppler width of which is independent of the number of trapped atoms.

When N or T is increasing, the behavior of a quantum system becomes more similar to that of the classical one. To get analytical formulas for the absorption line shape at arbitrarily large values of N and T, we shall make use of the semiclassical Thomas-Fermi (TF) approximation. In the TF approximation, the state of an atom is labeled by the radius vector $\mathbf{r}$ and wave vector $\mathbf{k}=\mathbf{p}/\hbar$ (see, e.g., Refs. [11,12]). The density of such states in the six-dimensional phase space $(\mathbf{r},\mathbf{k})$ is $(2\pi)^{-3}$. The number density of the fermion atoms in the phase space is

$$\rho(\mathbf{r},\mathbf{k},T)=\frac{1}{(2\pi)^3}\frac{1}{\exp\{\beta H_g(\mathbf{r},\hbar\mathbf{k})-\beta\mu\}+1}, \tag{7}$$

where the chemical potential μ is related to the number of trapped atoms via the normalization condition $\int d^3\mathbf{r}d^3\mathbf{k}\rho(\mathbf{r},\mathbf{k},T)=N$.

Applying the TF approximation, the line shape reads

$$I(\omega)=\sum_{\mathrm{ex}}|d_{\mathrm{ex}}|^2\int d^3\mathbf{r}d^3\mathbf{k}\rho(\mathbf{r},\mathbf{k},T)\,\delta(\omega-\omega_{\mathbf{r},\mathbf{k}}), \tag{8}$$

where $\omega_{\mathbf{r},\mathbf{k}}=M(\Omega_{\mathrm{ex}}^2-\Omega_g^2)(x^2+\lambda_y^2y^2+\lambda_z^2z^2)/2\hbar+\hbar k_x\kappa/M+\hbar\kappa^2/M+\omega_{0,\mathrm{ex}}$ is the transition frequency. If $\Omega_g=\Omega_{\mathrm{ex}}$, the frequency $\omega_{\mathbf{r},\mathbf{k}}$ does not depend on the atomic position $\mathbf{r}$, so the line shape is determined exclusively by the momentum distribution function:

$$n(\mathbf{k},T)=\int d^3\mathbf{r}\rho(\mathbf{r},\mathbf{k},T). \tag{9}$$

For $T=0$ the distribution function is given by [11]

$$n(\mathbf{k},0)=\frac{8N}{\pi^2K_F^3}\left(1-\frac{k^2}{K_F^2}\right)^{3/2}, \tag{10}$$

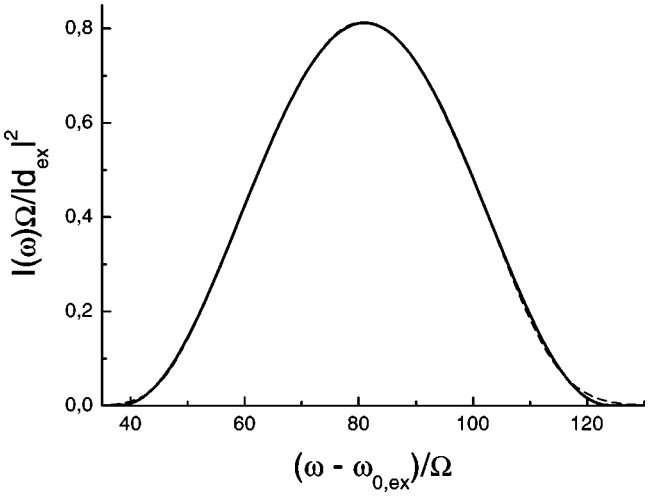

FIG. 2. Absorption line shape at $T=0$ calculated using the exact summation (dashed line) and the TF approximation (solid line) for $\alpha=9$, $N=35$, and $\lambda_y=\lambda_z=\lambda=1$.

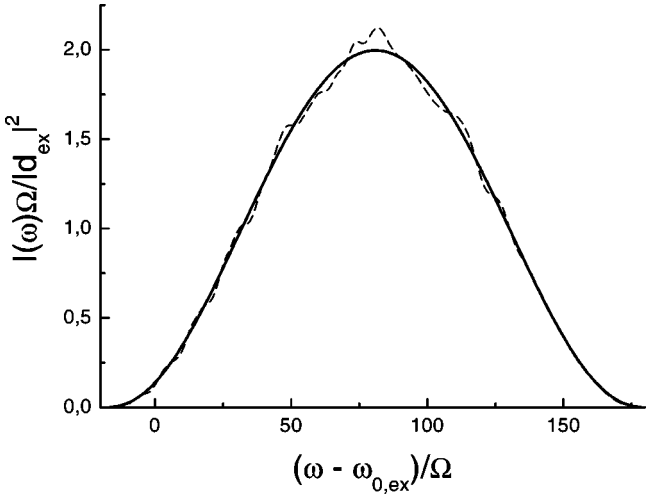

FIG. 3. The same as in Fig. 2, but for $N=196$ and $\lambda=5$.

where $K_F=(2ME_F/\hbar^2)^{1/2}$ is the maximum momentum of the trapped Fermi atoms at zero temperature, and $E_F=\hbar\Omega_g(6\lambda_y\lambda_z N)^{1/3}$ is the Fermi energy. Note that the momentum distribution is isotropic even though the trap is anisotropic [11]. This leads to an isotropic absorption line shape in the case where $\Omega_{ex}=\Omega_g$. Applying the distribution function (10), one arrives at the $(1-z^2)^{5/2}$ behavior of the line shape if $\Omega_{ex}=\Omega_g=\Omega$ and $T=0$;

$$I(\omega)=\frac{16N}{5\pi\Delta}\sum_{ex}|d_{ex}|^2[1-(\omega-\omega_{max})^2/\Delta^2]^{5/2}, \quad (11)$$

where the central frequency $\omega_{max}=\omega_{0,ex}+\alpha^2\Omega$ is shifted by the recoil frequency $\omega_{rec}=\alpha^2\Omega$ as compared to $\omega_{0,ex}$,

$$\Delta=2\alpha\Omega(6N\lambda_y\lambda_z)^{1/6} \quad (12)$$

being the spectral half-width. In the experiment by DeMarco and Jin [2] using trapped ^{40}K atoms, $\Omega=2\pi\times 19$ Hz, $N=7\times10^5$, and $\lambda\approx7$, giving $\alpha\approx36$ for $\omega=4\times10^{16}$ Hz. Consequently one has $\Delta\approx6\times10^5$ Hz. This is less than the typical radiative linewidths for free atoms. Yet, for trapped fermions the spontaneous emission is suppressed [7], so the Doppler broadening can be dominant.

The approximated line shape (11) depends on the trap anisotropy exclusively through the characteristic frequency $\Omega_{char}=\Omega(\lambda_y\lambda_z)^{1/3}$, which is a measure of the trap hardness. The bigger Ω_{char} is, the tighter is the trap, and the broader is the absorption spectrum. In fact, the maximum momentum of the atoms is larger in tighter traps (for the same number of trapped atoms) leading to the increase in the Doppler broadening. For instance, compared to an isotropic trap ($\lambda_y=\lambda_z=1$), the spectrum of a squeezed trap ($\lambda_y,\lambda_z>1$) is broader.

Exact and approximated line shapes are presented in Figs. 2 and 3. For an isotropic trap (Fig. 2), the agreement appears to be very good, even though the number of atoms N is rather small. Deviations are seen only in the tails of the spectrum corresponding to the periphery of the fermion cloud. In such an area, the fermion density becomes small and the TF approximation fails [11]. For anisotropic traps, the exact spectrum undergoes some oscillations about the approximated one even for relatively large values of N (see Fig. 3).

Consider next the situation where $T\neq0$ and the frequencies Ω_g and Ω_{ex} are not necessarily equal. The line shape (8) takes then the form for an isotropic trap:

$$I(\omega,T)=\sum_{ex}\frac{|d_{ex}|^2}{16\pi\alpha^6\Omega^6p^{5/2}}\int_0^\infty y^2dy\,\ln\{1+\exp[\beta\mu-y^2-p(\omega-\omega_{max}+m_{ex}y^2/\beta\hbar)^2]\}, \quad (13)$$

where $m_{ex}=1-\Omega_{ex}^2/\Omega_g^2$, $p=\beta\hbar/4\alpha^2\Omega$, and $\Omega\equiv\Omega_g$. If $\Omega_g=\Omega_{ex}$, the result (13) can be extended readily to anisotropic traps. In such a case, the line shape (13) acquires an extra factor $1/\lambda_y\lambda_z$, and the chemical potential μ depends on $\lambda_y\lambda_z$, in addition to T and N.

We are interested primarily in the strongly degenerate Fermi gas ($\beta E_F\gg1$), for which the Sommerfeld expansion holds for the chemical potential [11]:

$$\mu=E_F\left(1-\frac{\pi^2}{3}\left(\frac{1}{\beta E_F}\right)^2\right) \quad (14)$$

In the opposite (nondegenerate gas) limit ($\beta E_F\to0$), one has $\mu=\beta^{-1}\ln[(\beta E_F)^3/6]$, and the line shape (13) reduces to the Gaussian form if $\Omega_g=\Omega_{ex}$.

Figure 4 shows the temperature dependence of the line shapes for $\Omega_g=\Omega_{ex}$. At very low temperature ($kT/E_F=0.1$), the absorption spectrum is seen to be close to the zero-temperature limit. As the temperature increases, the absorption line shape becomes broader and is no longer characterized by the $(1-z^2)^{5/2}$ behavior. Yet, the line shape is still non-Gaussian, since the gas is strongly degenerate. Figure 5 shows the absorption spectrum for various values of $m_{ex}=1-\Omega_{ex}^2/\Omega_g^2$. For $\Omega_{ex}>\Omega_g$, the maximum position of the spectrum is shifted to larger frequencies. For $\Omega_{ex}<\Omega_g$, one has the opposite. Furthermore, one can see the obvious increase in the spectrum width if $\Omega_{ex}>\Omega_g$. This is due to the fact that an increase in the translational frequency of the

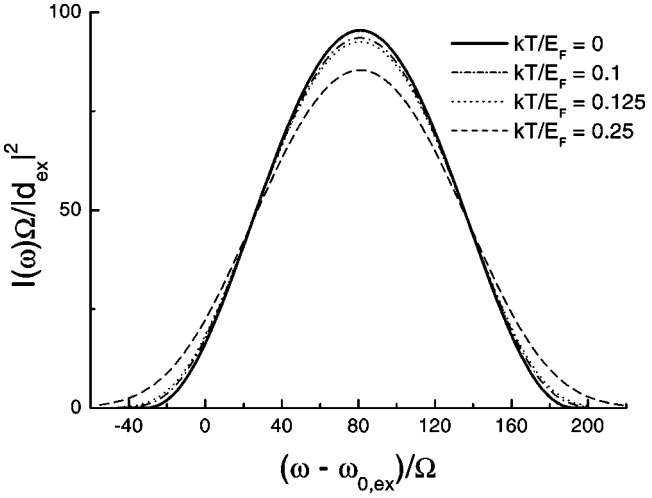

FIG. 4. Absorption line shape at various temperatures for an isotropic trap with $\Omega_g=\Omega_{\mathrm{ex}}$, $N=10667$, and $\alpha=9$.

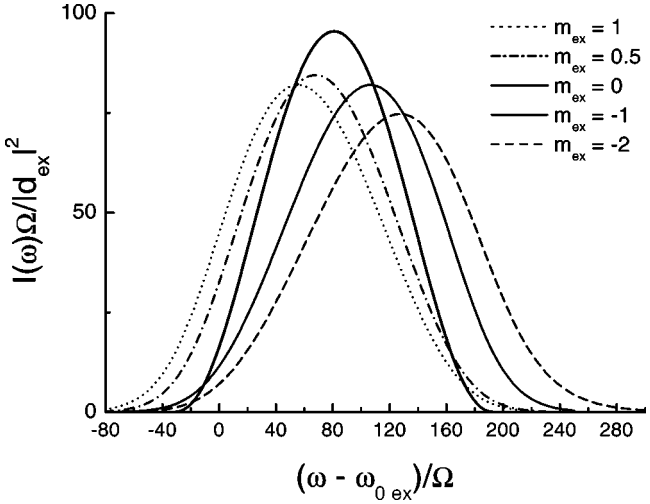

FIG. 5. Absorption line shape for an isotropic trap with $N=10\,667$, $\alpha=9$, $k_BT=0.25E_F$, and various $\Omega_{\mathrm{ex}}/\Omega_g$.

electronically excited atoms leads to an increase and broader distribution of frequencies of the optical transitions.

In summary, we have studied the absorption spectrum by a cold gas of Fermi atoms using both the exact summation and also the TF approximation. Oscillations have been obtained in the absorption line shape calculated exactly for the 1D and anisotropic 3D traps at a sufficiently small number of trapped atoms and zero temperature. No such oscillations appear for the isotropic 3D traps. Applying the TF approximation, relatively simple analytical expressions have been obtained for the line shape of 3D traps at a sufficiently large number of trapped atoms. At $T=0$, the approximated spectrum is characterized by a $(1-z^2)^{5/2}$ dependence. At nonzero temperature, the spectrum becomes broader, although it remains non-Gaussian as long as the fermion gas is degenerate. The changes in the trap frequency for an electronically excited atom can introduce an additional line broadening.

Note added in proof. Quantum degeneracy of lithium-6 fermions has recently been reported [20].

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