Filled Landau levels in neutral quantum gases

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We consider the signatures of the integer quantum Hall effect in a degenerate gas of electrically neutral atomic fermions. An effective magnetic field is achieved by applying two incident light beams with a high orbital angular momentum. We show how states corresponding to completely filled Landau levels are obtained and discuss various possibilities to measure the incompressible nature of the trapped two-dimensional gas.

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I. INTRODUCTION

Recent experimental advances in trapping and cooling atoms have enabled us to control and engineer the quantum states of delicate quantum gases such as the degenerate Fermi gases and Bose-Einstein condensates [1–4]. Ultracold atomic gases have turned out to be a remarkably good medium for studying a wide range of physical phenomena. This is mainly due to the fact that it is relatively easy to experimentally manipulate parameters of the system, such as the strength of interaction between the atoms, the properties of a lattice in which the atoms are trapped, the geometry of an external trap etc. Such a freedom of manipulating the parameters is usually not accessible in other systems known from condensed matter or solid state physics. In the present paper we study trapped spin-polarized fermions. Using fermions we naturally have a situation that closely resembles the electronic case with one important exception: the atoms are electrically neutral, and there is no vector potential term due to a magnetic field acting on the atoms. Therefore a direct analogy between the atomic and electronic cases is not necessarily straightforward.

The idea of producing *effective* magnetic fields in quantum gases has been investigated by several authors usually in connection with optical lattices [5–8] and external rotation [9,10]. It has recently been realized that in certain situations, in particular in a rotating frame, trapped atomic quantum gases can be used to recreate the physical state corresponding to filled Landau levels, and consequently quantum Hall states [9–15]. Present experimental techniques used for reaching filled Landau levels involve stirring of a Bose-Einstein condensate [9,10]. The ultimate goal here is to reach the state containing as many vortices in the superfluid as there are atoms. Experimentally it is, however, a rather demanding task to accurately control the stirring.

In recent papers [16,17] we have shown how to create an effective magnetic field without stirring, using two light beams (to be referred to as the control and probe beams) where at least one of them carries an orbital angular momentum. The effective magnetic field stems from the interaction of the laser beams with a medium of three-level atoms in the electromagnetically induced transparency (EIT) configuration (see Fig. 1). There is a significant advantage in creating the effective magnetic field using light, since the key to the form of the magnetic field lies in the phase and intensity of the light, concepts which with recent holographic techniques can be tailored to a remarkable degree nowadays. We are therefore now in a situation where we can choose different types of vector potentials and study their influence on quantum gases for both fermions and bosons.

In the present paper we investigate the physical properties of a degenerate two-dimensional Fermi gas of atoms in the presence of an effective magnetic field. We start by introducing the concept of a vector potential and the light-matter coupling. Subsequently we consider a trapped degenerate Fermi gas and calculate the effects of a strong magnetic field. Finally we conclude by discussing the experimental implications and some future prospects.

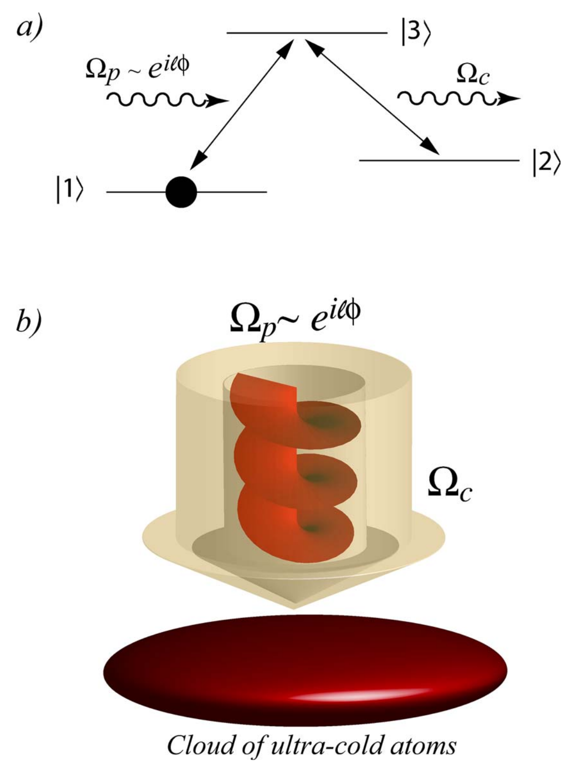

FIG. 1. (Color online) (a) The level scheme for electromagnetically induced transparency with the probe beam and control beams characterized by the Rabi frequencies Ω_p and Ω_c. (b) The experimental setup with two copropagating light beams and a cloud of cold atoms.

II. THE MODEL

Let us consider a neutral cloud of three-level atoms interacting with two incident beams of light: a probe beam containing an orbital angular momentum and a uniform control beam. The atoms are characterized by two hyperfine ground states 1 and 2 and an excited electronic state 3 (see Fig. 1). The two laser beams drive the atoms to the dark state, $|D\rangle \sim \Omega_c|1\rangle - \Omega_p|2\rangle$, representing a coherent superposition of the two hyperfine ground states 1 and 2. The corresponding equation for the wave function Φ_D representing the translational motion of the dark-state atoms is derived in Ref. [17],

$$i\hbar\partial_t\Phi_D = \frac{1}{2m}(i\hbar\nabla + \mathbf{A}_{\text{eff}})^2\Phi_D + V_{\text{eff}}(\mathbf{r})\Phi_D, \quad (1)$$

where

$$\mathbf{A}_{\text{eff}} = \frac{i\hbar}{2}\frac{\zeta^*\nabla\zeta - \zeta\nabla\zeta^*}{1+|\zeta^2|} \equiv -\hbar\frac{|\zeta|^2}{1+|\zeta^2|}\nabla S \quad (2)$$

and

$$V_{\text{eff}}(\mathbf{r}) = V_{\text{ext}}(\mathbf{r}) + \frac{\hbar^2}{2m}\frac{|\zeta|^2(\nabla S)^2 + (\nabla|\zeta|)^2}{(1+|\zeta|^2)^2} \quad (3)$$

are the *effective vector potential* and *effective trapping potential*. The external trapping potential for the dark-state atoms is given by

$$V_{\text{ext}}(\mathbf{r}) = \frac{V_1(\mathbf{r}) + |\zeta|^2(V_2(\mathbf{r}) + \epsilon_{21})}{1+|\zeta|^2} \quad (4)$$

with V_j being the trapping potential for the atoms in the hyperfine state j $(j=1,2)$, and

$$\epsilon_{21} = \hbar(\omega_2 - \omega_1 + \omega_c - \omega_p) \quad (5)$$

is the energy of the two-photon detuning with $\hbar\omega_i$ the energies of the hyperfine states. The dimensionless function $\zeta = \Omega_p/\Omega_c \equiv e^{iS}|\zeta|$ denotes the ratio between the Rabi frequencies for the probe and control beams, where $S=(\mathbf{k}_p-\mathbf{k}_c)\cdot\mathbf{r} + \ell\phi$ is the relative phase of the two beams, $\mathbf{k}_p$ and $\mathbf{k}_c$ are the wave vectors, ℓ is the winding number of the probe beam, and ϕ is the azimuthal angle.

In this way, the incident light field will act as a vector potential as in Eq. (1). The appearance of $\mathbf{A}_{\text{eff}}$ is a manifestation of the Berry connection which is encountered in many different areas of physics [18–20]. If we choose the control and probe beams copropagating, with the probe beam having an orbital angular momentum $\hbar\ell$ per photon and the intensity of the form

$$|\zeta|^2 = \frac{\alpha_0(r/R)^2}{1-\alpha_0(r/R)^2}, \quad (6)$$

we obtain a uniform magnetic field in the z direction, $\mathbf{B} = -2\hbar\alpha_0\ell R^{-2}\hat{\mathbf{e}}_z$, with $\mathbf{A}_{\text{eff}} = -\hbar\alpha_0\ell r R^{-2}\hat{\mathbf{e}}_\phi$ and $\alpha_0 < 1$ a dimensionless parameter. In the following we will choose the harmonic trapping potentials $V_1(\mathbf{r})$ and $V_2(\mathbf{r})$ such that $V_{\text{eff}}(\mathbf{r}) = 0$ for $r<R$. In Ref. [17] it is illustrated how this can be achieved using external potentials which are approximately harmonic, resulting in V_{eff} being close to zero over a large region. In addition we assume here a steep barrier at $r=R$. Such barriers have recently been experimentally demonstrated using optical potentials [21].

The atoms can be safely considered noninteracting, since for spin-polarized fermions only weak p-wave scattering is present [1–3,22]. The corresponding single-particle states describing the trapped fermions are governed by the equation

$$\left\{\frac{\hbar^2}{2m}\left[-\nabla^2 + \left(\frac{\ell\alpha_0}{R^2}\right)^2 r^2 + 2i\left(\frac{\ell\alpha_0}{R^2}\right)\partial_\phi\right] + V_{eff}(r)\right\}\Phi_D = E\Phi_D. \quad (7)$$

III. LANDAU LEVELS

Let us assume we have a two-dimensional Fermi gas with the atomic motion confined to the xy plane. After rescaling the radial coordinate $r=xR$ and using the ansatz $\Phi_D = \xi(x)e^{iq\phi}$ we obtain the solution in the form of a confluent hypergeometric function

$$\xi(x) = x^{|q|}e^{-(|\ell|\alpha_0/2)x^2}{}_1F_1\left[\frac{1+|q|}{2} - \left(\frac{\epsilon}{4|\ell|\alpha_0} + \frac{q}{2}\right), |q| + 1; |\ell|\alpha_0 x^2\right] \quad (8)$$

where $\epsilon = (E-E_z)2mR^2/\hbar^2$ and E_z is the transverse ground-state energy. As such, Eq. (8) is rather intractable. We can, however, obtain analytical expressions for the eigenvalues in the limit $|\ell|\alpha_0 \gg 1$, where the energies are of the form

$$\epsilon_{n,q} = 2|\ell|\alpha_0(2n + |q| - q + 1) \quad (9)$$

with $n=0,1,2,\ldots$ and $q=\ldots -2,-1,0,1,2,\ldots$. This is indeed the Landau result. The Landau system is strictly defined for an untrapped gas, but for $|\ell|\alpha_0 \gg 1$, the boundary at $r=R$ has little effect on the energies [23]. Note that the energy levels in Eq. (9) are highly degenerate and are spaced by $4\ell\alpha_0$. These levels are equivalent to the Landau levels of the charged system. The eigenstates for the Landau states are of the form

$$\xi(x) = e^{iq\phi}x^{|q|}e^{-|\ell|\alpha_0 x^2/2}L_n^{|q|}(|\ell|\alpha_0 x^2) \quad (10)$$

where $L_n^{|q|}(|\ell|\alpha_0 x^2)$ is the Laguerre polynomials.

Using the corresponding magnetic length $\ell_c = R/\sqrt{2\alpha_0\ell}$ the magnetic flux becomes $N_\phi = R^2/\ell_c^2 = 2\alpha_0\ell$. The Fermi gas is therefore described by the completely filled lowest Landau level if the criterion $N=N_\phi$ is satisfied where N is the number of atoms. On the other hand, it should be noted that the value of N_ϕ and the degeneracy are also limited by the fact that we have a finite trap.

It is at this point important to realize that the winding number ℓ of the light beams can with present technologies be of the order of a few hundred [24,25], whereas the parameter α_0 is smaller than 1. A high optical orbital angular momentum is achieved by creating a highly charged optical vortex. There are many different techniques to create optical vortices. Highly charged optical vortices are typically made using spatial light modulators which act as a phase hologram, where the grating can be programmed to achieve the required phase of the light beam. This method is mainly limited by the pixel density in the hologram.

IV. CHARACTERISTICS OF THE FILLED LANDAU LEVEL GAS

We will illustrate the completely filled Landau level gas by looking at the static and dynamical phenomena arising due to the effective magnetic field. One important question in this respect is how to distinguish between a gas described by the completely filled lowest Landau level and a gas that occupies more than one Landau level. In the atomic case the situation is slightly more subtle compered to the normal quantum Hall situation with electrons since with noninteracting atoms the concept of resistivity is not necessarily a useful and well-defined one.

We start by calculating the single-particle mass current. In order to do this we have to use the correct form of the current operator [26] which now takes the form

$$J_k(x) = -\frac{i}{2}[\Psi^\dagger(D_k\Psi) - (D_k\Psi)^\dagger\Psi] \qquad (11)$$

where $D_k=\partial_k - iA^k_{\text{eff}}/\hbar$. Using the eigenstates in Eqs. (10) and (11) we obtain the current density

$$\mathbf{J}^q(x) = \xi_q^*\mathbf{J}\xi_q = C_q^2 x^{2q-1} e^{-\alpha_0\ell x^2}\left(\frac{q}{\alpha_0\ell} - x^2\right)\hat{\mathbf{e}}_\phi, \qquad (12)$$

where C_q is a normalization constant. The current is clearly zero at the central distance $r_\ell=\sqrt{|q|/\alpha_0\ell}$ and flows in opposite directions on either side of r_ℓ. The total current is consequently going to be zero.

In the spirit of the integer quantum Hall effect in a Corbino geometry we may ask ourselves, what happens if we add a potential linear in r of the form $V(r)=\beta r$? If $mR^2\beta\ell_c/\hbar^2(\ell\alpha_0)^2 \ll 1$ the solutions of Eq. (7) can be approximated by shifting the solutions in Eq. (10) by the factor $\beta/2\alpha_0\ell$. The resulting single-particle current density then takes the form

$$\mathbf{J}^q(x) = C_q^2\left(x - \frac{1}{2}\frac{\beta}{\alpha_0\ell}\right)^{2q} e^{-\alpha_0\ell(x-\beta/2\alpha_0\ell)^2}\frac{1}{x}\left(\frac{q}{\alpha_0\ell} - x^2\right)\hat{\mathbf{e}}_\phi \qquad (13)$$

where the current is no longer zero as in the previous case. The total current per particle in the ϕ direction and in the lowest Landau level becomes $\mathbf{J}^q_{\text{tot}}=\int dx\,\mathbf{J}^q(x)=\beta\sqrt{\alpha_0\ell}\Gamma[q+1/2]/(4q!)\approx\beta\sqrt{\alpha_0\ell}/(4\sqrt{q})$. This also shows that the velocity varies as $1/r$ since the single-particle state is centered at $r=r_\ell=\sqrt{2q}R/\sqrt{\alpha_0\ell}$; hence the flow is irrotational. On the level of our approximations the current does not depend on which Landau level is occupied. Therefore the current depends nontrivially on the particle number which manifests itself as a jump in the derivative with respect to particle number or magnetic field. In Fig. 2 we show the total current $\mathbf{J}_{\text{tot}}=\Sigma_{q=0}^{N-1}\mathbf{J}^q_{\text{tot}}=\sqrt{\alpha_0\ell}\beta\Gamma(N+1/2)/[2(N-1)!]\approx\beta\sqrt{\alpha_0\ell N}/2$ as a function of particle number N.

The physics of the lowest completely filled Landau level is in itself an interesting concept and shows some rather intriguing scenarios from both a fundamental and an experimental point of view [27]. One of the most important questions concerning the atomic quantum Hall state is what to measure. For a gas described by the completely filled Landau level, the density is going to be homogeneous, since the density is effectively built up by shifted Gaussians corresponding to the different angular momenta as can be seen in Eq. (8). This also means we have added an external potential in order to have $V_{\text{eff}}=0$. This is not necessary but makes the situation simpler and more intuitive. Consequently, if there is no probe beam carrying orbital angular momentum propagating through the gas, the atoms will be subject to an external harmonic trap and hence will show a density profile quadratic in r, clearly different from the filled Landau state (see inset in Fig. 3). From an experimentalist's point of view a direct observation of the density of the cloud in the trap is not necessarily the most convenient way of observing the atoms. Another possibility is to consider the free expansion of the cloud.

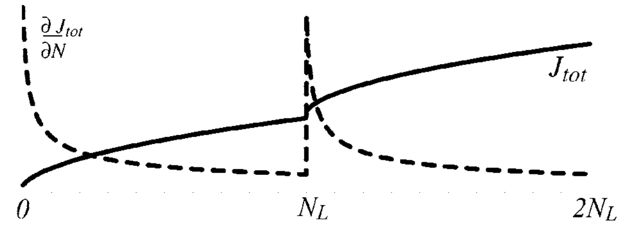

FIG. 2. The current as a function of particle number N and its derivative clearly show the transition between the Landau levels at $N=N_L$.

Since the trapped fermions can be considered noninteracting we can calculate the dynamics of the freely expanding cloud using the single-particle propagator,

$$K(r,r',\phi,\phi';t) = \frac{1}{i4\pi\tau}e^{(i/4\tau)[r^2+r'^2-2rr'\cos(\phi-\phi')]}, \qquad (14)$$

where τ is the rescaled time $t=(2mR^2/\hbar)\tau$ and r is in units of R. After a straightforward integration we obtain the dynamics of the freely expanding cloud using the states in Eq. (8) (see Fig. 3). The dynamics of the cloud described by the single completely filled Landau level is self-similar and is captured by the scaling parameter $\sigma_L=\sqrt{1+4\tau^2(\alpha_0\ell)^2}$. In Fig. 3 we show the mean width defined as $\Lambda(\tau)=2\sqrt{\int d\mathbf{r}\, r^2\rho(r)}/\sqrt{N}$ which can readily be calculated using the states in Eq. (8). The lowest completely filled Landau level is found to expand as $\Lambda(\tau)=\sigma_L(\tau)\sqrt{2(N+1)/\ell\alpha_0}$ [28].

Clearly the expansion of a single completely filled lowest Landau level compared to the situation with also the second Landau level filled is not going to be much different, since the expansion is still self-similar, apart from contributions from the edge states. The density of the trapped cloud is going to be homogeneous and will remain so when expanding. This is seen from the energy spectrum in Eq. (9). For an energy corresponding to the second Landau level we can have $n=1$ and $q\geq 0$ but also $n=0$ and $q=-1$. But this correction is only of the order of a single particle and will not be measurable; hence the expansion dynamics will be the same as in the lowest Landau level. There will, however, be a significant difference if we consider the two extreme situations with a Fermi gas described as a completely filled lowest Landau level and the situation when the effective mag-

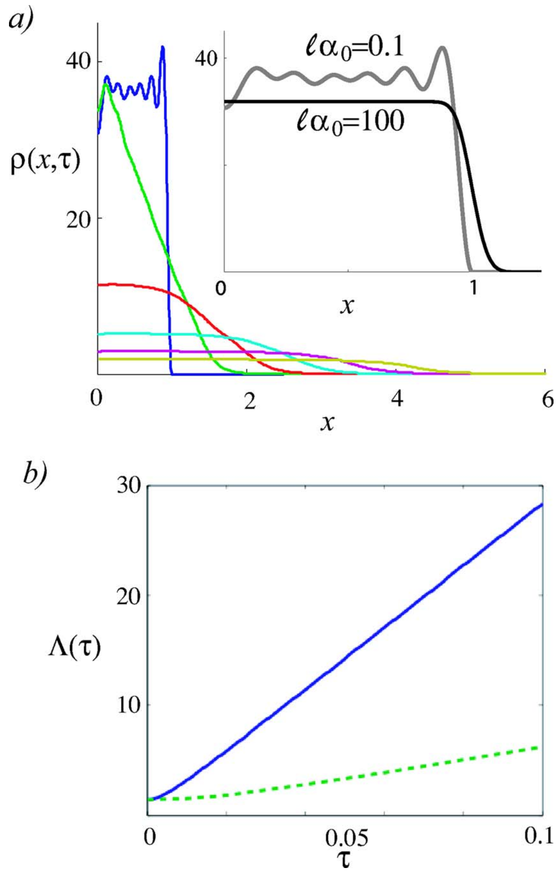

FIG. 3. (Color online) (a) The non-self similar expansion of the cloud of atoms corresponding to a weak magnetic field where many Landau levels are occupied ($\alpha_0\ell=0.1$) and $V_{\text{eff}}=0$. The different curves correspond to the times $\tau=0,0.02,0.04,0.06,0.08,0.10$ where τ is in units of $2mR^2/\hbar$. The inset shows a comparison between the densities at $\tau=0$ for $\ell\alpha_0=0.1$ which occupies many Landau levels and the cloud of atoms described by the single completely filled lowest Landau state, $N=\ell\alpha_0=100$ and $V_{\text{eff}}(x)=0$ for $x<1$. In both figures the total number of particles was $N=100$. (b) The diameter $\Lambda(\tau)=2\sqrt{\int d\mathbf{r}\, r^2\rho(r)}/\sqrt{N}$ for the completely filled Landau state is seen to expand much more rapidly than the state corresponding to a weak magnetic field ($\alpha_0\ell=0.1$).

netic field is weak. If the magnetic field is weak the trapped Fermi gas is well described by spherical Bessel functions corresponding to the eigenfunctions of a cylindrically trapped Fermi gas. The expansion dynamics can still be calculated with the free-particle propagator in Eq. (14). The important difference is evident in the expansion which is no longer going to be self-similar. Figure 3 shows the mean width of the cloud as a function of time compared to the completely filled lowest Landau level. The broken self-similarity is most clearly seen in a series of snapshots of the density compared to the density of the completely filled Landau level. The lowest Landau level states expand much faster than states belonging to higher Landau levels corresponding to the weak magnetic field case. This is easily understood since the lowest Landau level states are pure angular momenta states with no radial excitation, whereas the gas with a weak magnetic field contains states with radial excitations but lower angular momenta, and hence lower kinetic energy.

V. CONCLUSIONS

Throughout this paper we have considered a two-dimensional trapped Fermi gas. It is important to remember that a two-dimensional Fermi gas imposes some rather strict conditions on the external trap configuration. The two-dimensionality is preserved if the Fermi energy is lower than the relevant transverse ground-state energy. The relevant energy scale is here the effective cyclotron frequency which for typical radii of the cloud (a few tens of micrometers) can be of the order of 100 Hz; hence the transverse trap frequency needs to be significantly stronger than this. Such traps are indeed readily available [29]. The effective magnetic field relies on the stability of the dark state. As discussed in Ref. [17], with typical experimental parameters, the dark state will be stable for times significantly longer than the normal lifetime for a trapped cloud, making the effective magnetic field created by optical orbital angular momentum a feasible technique for achieving strong magnetic fields. Clearly, from an experimental point of view, the biggest challenge is found in the detection of the mass current in the cloud. There are, however, powerful techniques based on slow light propagation and the dragging of the light [26,30] which will identify a mass current in the cloud *in situ*. Another possibility is to measure the shape oscillations which should be affected by the current.

In this paper we have investigated the concept of completely filled Landau levels in trapped Fermi gases. The effective magnetic field was created using light with orbital angular momentum. The recent advances in creating exotic light beams where both phase and intensity can be manipulated, allows us to consider many different forms of the effective magnetic field. We have restricted ourselves to the "textbook" scenario with a homogeneous effective magnetic field. It is important to note here that this scenario is different from other techniques where filled Landau levels are considered. In the case of bosons in a harmonic trap, the gas is stirred to create many vortices resulting in an angular momentum which would correspond to the trap ground-state energy. In our case the situation is much simpler. Only a static external trap needs to be added which matches the corresponding cyclotron frequency. If the frequency of the added harmonic trap does not match the cyclotron frequency, the important degeneracy will be lifted, but as long as

$$N = q_{\max} < \frac{2}{1 - 1/\sqrt{1 + (\Delta\omega/\omega_c)^2}} \approx 4\left(\frac{\omega_c}{\Delta\omega}\right)^2 \qquad (15)$$

where $\omega_c=\hbar\alpha_0\ell/(mR^2)$ and $\Delta\omega$ is the preferably small deviation, the atoms that fill the lowest Landau level will not mix with the higher levels.

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