

Slow polaritons with orbital angular momentum in atomic gases

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(Received 14 May 2010; published 17 February 2011)

Polariton formalism is applied for studying the propagation of a probe field of light in a cloud of cold atoms influenced by two control laser beams of larger intensity. The laser beams couple resonantly three hyperfine atomic ground states to a common excited state, thus forming a tripod configuration of the atomic energy levels involved. The first control beam can have an optical vortex, with the intensity of the beam going to zero at the vortex core. The second control beam without a vortex ensures the lossless (adiabatic) propagation of the probe beam at a vortex core of the first control laser. We investigate the storage of the probe pulse into atomic coherences by switching off the control beams and its subsequent retrieval by switching the control beams on. The optical vortex is transferred from the control to the probe fields during the storage or retrieval of the probe field. We analyze the conditions needed for the vortex to be transferred efficiently to the regenerated probe beam and discuss the possibilities of experimental implementation of the proposed scheme using atoms such as rubidium or sodium.

DOI: [10.1103/PhysRevA.83.023812](https://doi.org/10.1103/PhysRevA.83.023812)

PACS number(s): 42.50.Gy, 03.67.–a, 42.50.Tx

I. INTRODUCTION

During the last several years, there has been a great deal of interest in slow [1–5], stored [6–18], and stationary [19–25] light. Light can be slowed down by 7 orders of magnitude to velocities of several tens of meters per second [1] due to the electromagnetically induced transparency (EIT) [26–30]. The EIT makes a resonant and opaque medium transparent for a probe beam by applying a control laser beam of larger intensity. The probe beam couples resonantly the ground and excited atomic states, whereas the control beam couples the same excited state to another unpopulated atomic ground state. This makes a Λ configuration of the atomic energy levels involved, as depicted in Fig. 1. The optical transitions induced by both laser beams interfere destructively, preventing population of the excited atomic state. As a result, a weak pulse of probe light travels slowly and with little losses in a resonant medium due to the application of the control laser beam.

The EIT was shown not only to slow down dramatically light pulses [1–5], but also to store them [7,8,13,15–18] in atomic gases. The storage and release of a probe pulse has been accomplished [7,8,13,15–18] by switching off and on the control laser [6]. The coherent control of the propagation of quantum light pulses can lead to a number of applications, such as generation of nonclassical states in atomic ensembles and reversible quantum memories for slow light [6,9,10,13,29–33]. On the other hand, propagation of slow light through moving media [34–41] may be used for the light memories and rotational sensing devices.

The orbital angular momentum (OAM) [42,43] provides a new element to the slow light, giving additional possibilities in manipulation of the optical information during the storage and retrieval of the slow light. The previous studies have concentrated on situations where the probe beam contains an OAM [41,44–47]. In this paper, we consider another scenario in which a control laser beam can carry an optical vortex. The intensity of such a control beam goes to zero at the vortex core, leading to the absorption losses of the probe beam in this area. To avoid the losses, we suggest the use of an additional control laser without an optical vortex, so that the total intensity of the control lasers is nonzero at the vortex core of the first

control laser. The probe along with both control laser fields induce transitions between the atomic energy levels in a tripod configuration of the light-atom coupling [48–59], as depicted in Fig. 2(a). We show that the regenerated slow light can acquire the OAM if one of the control beams contains it. We explore conditions for the optical vortex of the control beam to be transferred efficiently to the regenerated probe beam.

The tripod scheme can be realized for atoms such as sodium [7] or rubidium [8] containing two hyperfine ground levels with $F = 1$ and 2, as depicted in Fig. 2(c). These atoms have been employed in the original experiments on the storage of slow light based on a simpler Λ setup [7,8]. In this situation, $|1\rangle$ and $|3\rangle$ correspond to the magnetic sublevels (with $m_F = 1$ and -1) of the $F = 1$ hyperfine ground level, whereas the state $|3\rangle$ represents the hyperfine ground state with $F = 2$ and $m_F = 1$. The probe beam is to be σ^+ polarized, whereas both control beams are to be σ^- polarized to make a tripod setup. Such a scheme can be produced by adding an extra circularly polarized laser beam Ω_{c3} as compared to the experiment by Liu *et al.* [7] on the light storage in the sodium gases using the Λ scheme. Thus, it is feasible to implement the suggested experiment on the transfer of optical vortex from the control to the probe fields using the tripod setup.

The storage and retrieval of slow light is analyzed using the polariton formalism. The starting point is a set of the atomic equations together with the equation for the probe field. Subsequently, we obtain two coupled equations for dark-state polaritons representing the slow light in the atomic medium. We provide conditions for when the polaritons are decoupled. An advantage of polariton formalism is the simplicity of the relationship between the polariton field and the regenerated electric field, a feature that is missing in the direct analysis of the probe beam propagation [59]. Furthermore, the equation for the polariton has a usual form of matter wave equation, which describes the atomic evolution when the control fields are off.

II. INITIAL EQUATIONS

We will deal with an ensemble of atoms characterized by three hyperfine ground states $|1\rangle$, $|2\rangle$, and $|3\rangle$, as well as an

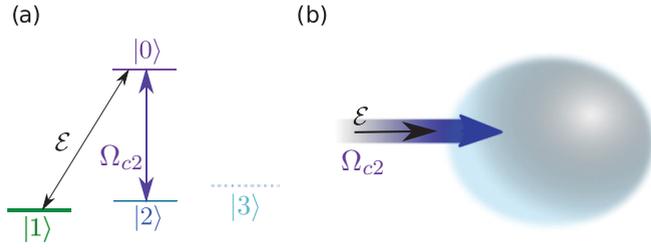


FIG. 1. (Color online) Probe and control laser beams acting on atoms characterized by two hyperfine ground states $|1\rangle$ and $|2\rangle$ as well as an excited state $|0\rangle$ to form a three-level scheme of the Λ type. Atoms are initially in the ground state $|1\rangle$. Stimulated exchange of photons between the probe (\mathcal{E}) and control (Ω_{c2}) laser fields creates a superposition of the hyperfine atomic ground states $|1\rangle$ and $|2\rangle$, making the medium transparent for the resonant probe pulse.

electronic excited state $|0\rangle$. The atomic internal and center-of-mass dynamics is represented by a four-component field $\Psi(\mathbf{r})$. Its components $\Psi_1(\mathbf{r}, t)$, $\Psi_2(\mathbf{r}, t)$, $\Psi_3(\mathbf{r}, t)$, and $\Psi_0(\mathbf{r}, t)$ describe the atomic center-of-mass motion in the corresponding internal states $|1\rangle$, $|2\rangle$, $|3\rangle$, and $|0\rangle$. In the semiclassical (mean-field) approach, $\Psi_j(\mathbf{r}, t)$ defines the probability amplitude to find an atom positioned at \mathbf{r} in the j th internal state, with $j = 0, 1, 2, 3$. In the fully quantum approach, $\Psi_j(\mathbf{r}, t)$ is the corresponding field operator.

Three beams of light act on the atoms in a tripod configuration of the atom-light coupling [48–59]. Two strong classical control lasers induce transitions $|2\rangle \rightarrow |0\rangle$ and $|3\rangle \rightarrow |0\rangle$, whereas a weaker probe field drives a transition $|1\rangle \rightarrow |0\rangle$, as shown in Fig. 2. The former control lasers are characterized by the Rabi frequencies Ω_{c2} and Ω_{c3} to be treated as incident variables. The latter probe beam is a dynamical quantity described by the electric field strength

$$\mathbf{E}(\mathbf{r}, t) = \hat{\mathbf{e}} \sqrt{\frac{\hbar\omega}{2\epsilon_0}} \mathcal{E}(\mathbf{r}, t) e^{-i\omega t} + \text{H.c.}, \quad (1)$$

where $\omega = ck$ is the central frequency of the probe photons, $\mathbf{k} = \hat{\mathbf{z}}k$ is the wave vector, and $\hat{\mathbf{e}} \perp \hat{\mathbf{z}}$ is the unit polarization vector. The probe field can be treated either as a classical variable or as a quantum operator. The dimensions of the electric field amplitude \mathcal{E} are chosen such that its squared modulus represents a number density of probe photons.

The probe field is considered to be quasimonochromatic, and its amplitude $\mathcal{E} \equiv \mathcal{E}(\mathbf{r}, t)$ changes little over the optical

cycle. The slowly (in time) varying amplitude of the probe field obeys the following equation:

$$\left(\frac{\partial}{\partial t} - i \frac{c^2}{2\omega} \nabla^2 - i \frac{\omega}{2} \right) \mathcal{E} = ig \Phi_1^* \Phi_0, \quad (2)$$

where the parameter $g = \mu\sqrt{\omega/2\epsilon_0\hbar}$ characterizes the strength of coupling of the probe field with the atoms, with μ being the dipole moment of the atomic transition $|1\rangle \rightarrow |0\rangle$. The quantities on the right-hand side of Eq. (2), i.e., Φ_0 and Φ_1^* , represent slowly (in time) varying atomic fields. The asterisk in Φ_1^* refers either to the complex conjugation of a classical field or the Hermitian conjugation of a quantum field. The slowly varying atomic fields Φ_j ($j = 1, 2, 3, 4$) are related to the original ones as $\Phi_1 = \Psi_1 e^{i\omega_1 t}$, $\Phi_2 = \Psi_2 e^{i(\omega_1 + \omega - \omega_{c2})t}$, $\Phi_3 = \Psi_3 e^{i(\omega_1 + \omega - \omega_{c3})t}$, $\Phi_0 = \Psi_0 e^{i(\omega_1 + \omega)t}$, where $\hbar\omega_1$ is the energy of the atomic ground state 1, whereas ω_{c2} and ω_{c3} are the frequencies of the control fields.

The atomic equations of motion read as

$$\hat{K} \Phi_1 = V_1(\mathbf{r}) \Phi_1 - \hbar g \mathcal{E}^* \Phi_0, \quad (3)$$

$$\hat{K} \Phi_0 = \hbar(\omega_{01} - i\gamma) \Phi_0 + V_0(\mathbf{r}) \Phi_0 - \hbar \Omega_{c2} \Phi_2 - \hbar \Omega_{c3} \Phi_3 - \hbar g \mathcal{E} \Phi_1, \quad (4)$$

$$\hat{K} \Phi_2 = \hbar \omega_{21} \Phi_2 + V_2(\mathbf{r}) \Phi_2 - \hbar \Omega_{c2}^* \Phi_0, \quad (5)$$

$$\hat{K} \Phi_3 = \hbar \omega_{31} \Phi_3 + V_3(\mathbf{r}) \Phi_3 - \hbar \Omega_{c3}^* \Phi_0, \quad (6)$$

with

$$\hat{K} = i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2}{2m} \nabla^2, \quad (7)$$

where $\omega_{21} = \omega_2 - \omega_1 + \omega_{c2} - \omega$ and $\omega_{31} = \omega_3 - \omega_1 + \omega_{c3} - \omega$ are the frequencies of the electronic detuning from the two-photon resonances, $\omega_{01} = \omega_0 - \omega_1 - \omega$ is the frequency of the electronic detuning from the one-photon resonance, and γ is the decay rate of the excited electronic level. Note that the inclusion of the nonzero decay rates should be generally accompanied by the introduction of the noise operator in the equations of motion [28]. Yet, in the present situation, one can disregard the latter noise: we are working in the linear regime with respect to the probe field, so the population of the excited state is small enough. In addition, m is the atomic mass and $V_j(\mathbf{r})$ is the trapping potential for an atom in the internal state j ($j = 1, 2, 3, 0$). The terms containing atomic mass m are important for the description of the light-dragging effects [34–41].

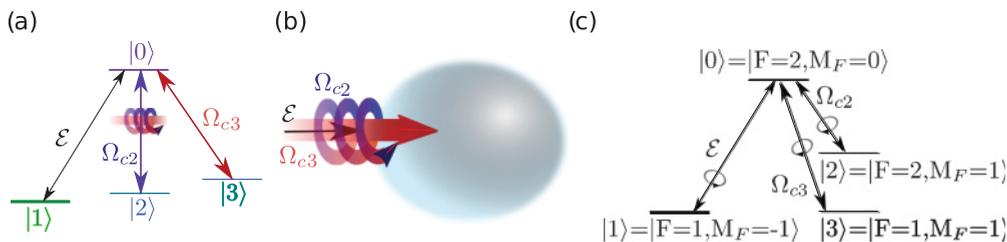


FIG. 2. (Color online) (a) Tripod scheme of the atom-light coupling involving a probe beam (\mathcal{E}) and two control beams (Ω_{c2} and Ω_{c3}). The three beams induce transitions between the atomic excited state $|0\rangle$ and three ground states $|1\rangle$, $|2\rangle$, and $|3\rangle$. (a), (b) A control beam with the Rabi frequency Ω_{c2} can have an optical vortex. Application of an additional control laser beam without the OAM ($\Omega_{c3} \neq 0$) makes it possible to avoid losses in the propagation of the probe beam at the vortex core, where the amplitude Ω_{c2} goes to zero. (c) A possible experimental realization of the tripod setup for atoms such as sodium [7] or rubidium [8] containing the hyperfine ground states with $F = 1$ and 2.

In Eqs. (3)–(6), the coupling of atoms with the probe and control fields has been written using the rotating wave approximation. Therefore, the last term in Eq. (3) has a negative frequency part of the probe field (\mathcal{E}^*), whereas the last term in Eq. (4) has a positive frequency part (\mathcal{E}). Similarly, Eq. (4) contains Rabi frequencies Ω_{c2} and Ω_{c3} , whereas Eqs. (5) and (6) contain their complex-conjugated counterparts Ω_{c2}^* and Ω_{c3}^* .

The equation of motion (3) for Φ_1 does not explicitly accommodate collisions between the ground-state atoms. If the atoms in the internal ground state 1 form a Bose-Einstein condensate (BEC), the collisional effects can be included, replacing $V_1(\mathbf{r})$ by $V_1(\mathbf{r}) + g_{11}|\Phi_1|^2$ in Eq. (3), where $g_{11} = 4\pi\hbar^2 a_{11}/m$ and a_{11} is the scattering length between the condensate atoms in the internal state 1. This yields a mean-field equation for the condensate wave function Φ_1 .

Initially, the atoms populate the ground level 1. We are interested in the linear regime where the modulus of Rabi frequency of the probe field $\Omega_p = g\mathcal{E}$ is much smaller than the total Rabi frequency of the control beams Ω_c given by Eq. (9) below. Consequently, one can neglect the last term in Eq. (3) that causes depletion of the ground level 1. This provides a closed equation for the ground-state dynamics $\hat{K}\Phi_1 = V_1(\mathbf{r})\Phi_1$. If the atoms in the internal ground state 1 form a BEC, its wave function $\Phi_1 = \sqrt{n} \exp(iS_1)$ represents an incident variable determining the atomic density n and the condensate phase S_1 . The latter phase will not play an important role in our subsequent analysis, since we are not interested in the influence of the condensate dynamics on the propagation of slow light. The phase will be taken to be zero ($S_1 = 0$) when dealing with the storage and retrieval of slow light in Sec. IV.

III. DARK- AND BRIGHT-STATE POLARITONS

When the probe photons enter the atomic media, they are converted into composite quasiparticles of the radiation and atomic excitations known as polaritons. Let us first introduce the bright-state polariton Φ_B :

$$\Phi_B = \zeta_c(\xi_{c2}\Phi_2 + \xi_{c3}\Phi_3) + \zeta_1\mathcal{E}, \quad (8)$$

where

$$\Omega_c = \sqrt{|\Omega_{c2}|^2 + |\Omega_{c3}|^2} \quad (9)$$

is the total Rabi frequency,

$$\xi_{c2} = \Omega_{c2}/\Omega_c, \quad \xi_{c3} = \Omega_{c3}/\Omega_c, \quad (10)$$

$$\zeta_1 = g\Phi_1/\Xi, \quad \zeta_c = \Omega_c/\Xi \quad (11)$$

are dimensionless parameters, and

$$\Xi = \sqrt{\Omega_c^2 + g^2 n}. \quad (12)$$

The polariton Φ_B represents a specific superposition of the atomic and the probe fields featured in the equation of motion (4) for atoms in the excited electronic state. The latter equation (4) can be rewritten in terms of the bright-state polariton

$$\hat{K}\Phi_0 = \hbar(\omega_{01} - i\gamma)\Phi_0 + V_0(\mathbf{r})\Phi_0 - \hbar\Xi\Phi_B. \quad (13)$$

In this way, the bright-state polariton is responsible for the light-induced atomic transitions to the excited state.

The two dark-state polaritons are defined as superpositions of the atomic coherences and the probe photons orthogonal to the bright-state polariton Φ_{B1} :

$$\Phi_{D1} = \zeta_c\mathcal{E} - \zeta_1^*(\xi_{c2}\Phi_2 + \xi_{c3}\Phi_3), \quad (14)$$

$$\Phi_{D2} = \xi_{c3}^*\Phi_2 - \xi_{c2}^*\Phi_3. \quad (15)$$

It is to be noted that only the first dark-state polariton Φ_{D1} of the tripod scheme contains the electric probe field component and thus has a nonzero radiative group velocity. The incoming light is converted exclusively into this polariton when it enters the medium. The second dark-state polariton Φ_{D2} does not have any contribution by the probe photons and is thus characterized by a zero radiative group velocity. It corresponds to the dark state of the Λ system consisting of the levels 2, 3, and 0. The combination $\xi_{c2}\Phi_2 + \xi_{c3}\Phi_3$ featured in Eqs. (8) and (14) represents the bright state of such a Λ system. In this way, only the first polariton experiences the radiative motion, with the second one being trapped in the atomic medium.

The “bare” atomic and probe fields can be cast in terms of the dark and bright polaritons of the tripod system as

$$\Phi_2 = \xi_{c2}^*(\zeta_c\Phi_B - \zeta_1\Phi_{D1}) + \xi_{c3}\Phi_{D2}, \quad (16)$$

$$\Phi_3 = \xi_{c3}^*(\zeta_c\Phi_B - \zeta_1\Phi_{D1}) - \xi_{c2}\Phi_{D2}, \quad (17)$$

$$\mathcal{E} = \zeta_1^*\Phi_B + \zeta_c\Phi_{D1}. \quad (18)$$

To obtain the equation for the dark-state polaritons, one needs to take the time derivative of Eqs. (14) and (15) and make use of the equations of motion (2), (5), (6), and (13).

Suppose the control and probe beams are tuned close to the two-photon resonance. Application of such beams causes EIT, in which the transitions $|1\rangle \rightarrow |0\rangle$, $|2\rangle \rightarrow |0\rangle$, and $|3\rangle \rightarrow |0\rangle$ interfere destructively, preventing population of the excited state 0. As a result, the atom-light system is driven to the dark states, and the bright-state polariton Φ_B [featured in the equation of motion (13) for the excited state atoms] is weakly populated: $\Phi_B \approx 0$. Neglecting the contribution due to the bright-state polariton Φ_B (adiabatic approximation), one obtains the equations for the dark-state polaritons Φ_{D1} and Φ_{D2} . Introducing a column $\Phi = (\Phi_{D1}, \Phi_{D2})^T$, it is convenient to represent these equations in a matrix form

$$i\hbar \frac{\partial}{\partial t} \Phi = \left[-\frac{\hbar^2}{2} \begin{pmatrix} 1/m_{D1} & 0 \\ 0 & 1/m \end{pmatrix} \nabla^2 + i\hbar \mathbf{J} \cdot \nabla + U \right] \Phi, \quad (19)$$

where the 2×2 matrices \mathbf{J} and U are defined in Appendix A. The former \mathbf{J} represents a complex vector potential, with U being a complex scalar potential. Even though the potentials are complex, the equation of motion (19) is Hermitian and thus it preserves the norm of the column Φ . Also,

$$m_{D1} = \left(\frac{c^2}{\hbar\omega} |\zeta_c|^2 + \frac{1}{m} |\zeta_1|^2 \right)^{-1} \quad (20)$$

is the effective mass of the first dark-state polariton. The mass m_{D1} exhibits position and time dependence through its dependence on the Rabi frequencies of the control fields and also on the atomic density. The second polariton does not have

a radiative component, so its effective mass coincides with the atomic mass m in Eq. (19).

The effective mass of the first polariton can be represented as

$$m_{D1} = \left(\frac{1}{m_{\text{rad}}} + \frac{1}{m} \frac{g^2 n}{\Omega_c^2 + g^2 n} \right)^{-1}, \quad (21)$$

where

$$m_{\text{rad}} = \frac{\hbar\omega}{c v_{\text{rad}}} = m \frac{v_{\text{rec}}}{v_{\text{rad}}} \quad (22)$$

and

$$v_{\text{rad}} = \frac{c^2 \Omega_c^2}{\Omega_c^2 + g^2 n} \quad (23)$$

are, respectively, the radiative ‘‘mass’’ and the radiative group velocity of the first polariton, with $v_{\text{rec}} = \hbar\omega/mc$ being the atomic recoil velocity. In the slow light regime where $\Omega_c^2 \ll g^2 n$, the latter $v_{\text{rad}} \approx c^2 \Omega_c^2 / g^2 n$ is much smaller than the vacuum speed of light $v_{\text{rad}} \ll c$. The radiative velocity v_{rad} can be of the order of 10 m/s for the slow light in atomic gases [1]. This greatly exceeds the typical velocities associated with the center-of-mass motion of cold atoms. For instance, the atomic recoil velocity is typically of the order of 1 cm/s. Thus, the second term can be neglected in Eq. (21), giving $m_{D1} \approx m_{\text{rad}}$.

A. Co-propagating probe and control beams

Suppose that the control beams propagate along the z axis with $k_{c2} \approx k_{c3} = k_c$:

$$\Omega_{c2} = \Omega'_{c2} e^{ik_c z}, \quad \Omega_{c3} = \Omega'_{c3} e^{ik_c z}. \quad (24)$$

For paraxial control beams, the amplitudes Ω'_{c2} and Ω'_{c3} depend weakly on the propagation direction z . It is convenient to represent the dark-state polaritons as

$$\Phi_{D1}(\mathbf{r}, t) = \Phi'_{D1}(\mathbf{r}, t) e^{ik_c z}, \quad (25)$$

$$\Phi_{D2}(\mathbf{r}, t) = \Phi'_{D2}(\mathbf{r}, t) e^{-ik_c z}, \quad (26)$$

where the amplitudes $\Phi'_{D1}(\mathbf{r}, t)$ and $\Phi'_{D2}(\mathbf{r}, t)$ depend slowly on the propagation direction z in the paraxial case. Introducing a column $\Phi' = (\Phi'_{D1}, \Phi'_{D2})^T$ in Eq. (19) provides the following equation for the slowly varying amplitudes:

$$i\hbar \left[\frac{\partial}{\partial t} + \begin{pmatrix} v_{g1} & 0 \\ 0 & 0 \end{pmatrix} \frac{\partial}{\partial z} \right] \Phi' = \left[-\frac{\hbar^2}{2} \begin{pmatrix} \frac{1}{m_{D1}} & 0 \\ 0 & \frac{1}{m} \end{pmatrix} \nabla^2 + i\hbar \mathbf{J}' \cdot \nabla + U' \right] \Phi', \quad (27)$$

where the 2×2 matrices \mathbf{J}' and U' are presented in Appendix B. Here,

$$v_{g1} = v_{\text{rad}} + \frac{\hbar}{m} (k - k_c) |\zeta_1|^2 \quad (28)$$

is the group velocity of the first dark-state polariton. It comprises the radiative group velocity and the velocity of the two-photon recoil. The latter term can be neglected giving $v_{g1} \approx v_{\text{rad}}$.

B. Decoupled dark-state polaritons

Let us analyze the terms that couple both dark polaritons in the equation of motion (27). The term with time derivatives in the nondiagonal elements of the matrix U' is proportional to

$$\xi_{c2} \frac{\partial}{\partial t} \xi_{c3} - \xi_{c3} \frac{\partial}{\partial t} \xi_{c2} = \frac{\Omega_{c2}}{\Omega_c^2} \frac{\partial}{\partial t} \Omega_{c3} - \frac{\Omega_{c3}}{\Omega_c^2} \frac{\partial}{\partial t} \Omega_{c2}. \quad (29)$$

If both control pulses depend on time in the same manner, i.e., $\Omega_{c2} = \Omega_{c2}^{(0)} f(t)$ and $\Omega_{c3} = \Omega_{c3}^{(0)} f(t)$, the above term is zero. Thus, the coupling between the two dark-state polaritons can be avoided by switching both control pulses off and on in the same way so that both of them exhibit the same temporal behavior.

Let us next estimate nondiagonal terms that contain the spatial derivatives of the control pulses in the equation of motion (27) and, hence, couple both dark polaritons. Such nondiagonal matrix elements are of the order of the atomic recoil energy $\hbar\omega_{\text{rec}} = \hbar^2 k^2 / (2m)$ and, thus, can be neglected if the characteristic interaction time between the two dark-state polaritons $\tau_{\text{pulse}} = l/v_{g1}$ is small compared with the reciprocal recoil frequency $\omega_{\text{rec}} \tau_{\text{pulse}} \ll 1$, where l is the length of the probe pulse in the medium. The latter condition can be easily fulfilled for typical slow light pulses, the durations of which are of the order of a microsecond [1] and thus are much smaller than the reciprocal recoil frequencies. Consequently, the polaritons Φ_{D1} and Φ_{D2} are decoupled and equations for them can be solved separately.

We are interested in the equation for the first dark polariton. Such a polariton contains the radiative contribution and thus describes propagation of the probe pulse of light in the medium. Neglecting the coupling with the second polariton, Eq. (27) yields a closed equation for the paraxial propagation of the first polariton along the z direction:

$$i\hbar \left(\frac{\partial}{\partial t} + v_{g1} \frac{\partial}{\partial z} \right) \Phi'_{D1} = -\frac{\hbar^2}{2m_{D1}} \nabla^2 \Phi'_{D1} + i\hbar \mathbf{J}'_{11} \cdot \nabla \Phi'_{D1} + U'_{11} \Phi'_{D1}, \quad (30)$$

with $v_{g1} \approx v_{\text{rad}}$. Due to the finite lifetime of the excited atomic state γ^{-1} , the first polariton will experience radiative losses, which are not included in the propagation equations (27) and (30). Let us now estimate the losses. The polariton lifetime is determined by the rate of the excited-state decay and the total Rabi frequency of the control lasers Ω_c [9]: $\tau_{\text{pol}} = \gamma^{-1} (\Omega_c / \Delta\omega)^2$, where $\Delta\omega$ is a detuning from the two-photon resonance. One of the reasons for the appearance of the two-photon detuning is the finite duration of the probe pulse $\Delta\omega = \tau_{\text{pulse}}^{-1}$. To avoid the losses, the time in which the polariton transverses the sample should be smaller than the polariton lifetime: $L/v_{\text{rad}} \ll \tau_{\text{pol}}$, with L being the length of the atomic cloud. This means that the total Rabi frequency Ω_c should be large enough:

$$L \ll v_{\text{rad}} \gamma^{-1} \Omega_c^2 \tau_{\text{pulse}}^2. \quad (31)$$

Note also that, in the slow light regime, the probe radiation makes a tiny contribution to the polariton, which is composed predominantly of the atomic excitations (atomic coherences). In fact, the velocity ratio $v_{\text{rad}}/c \ll 1$ represents a fraction of the radiation component in the polariton [6,9]. Thus, Eq. (30)

effectively describes propagation of the atomic coherences along the z axis at the velocity $v_{\text{rad}} \ll c$ appearing due to the small radiative component.

IV. STORAGE AND RELEASE OF SLOW LIGHT: GENERAL

A. Storage of slow light

Let us first consider the storage of the slow light. The probe beam $\mathcal{E}^{(s)}$ enters the atomic medium at $z = z_0$. The medium is illuminated by two control beams characterized by Rabi frequencies $\Omega_{c2}^{(s)}$ and $\Omega_{c3}^{(s)}$, where the index (s) refers to the storing stage of light. Initially, the Rabi frequencies of the control beams (and hence the group velocity $v_{g1} \equiv v_{g1}^{(s)}$) are time independent. Neglecting the diffraction effects, one can thus write

$$\mathcal{E}^{(s)}(t, z) = \mathcal{E}^{(s)}(\tau, z_0), \quad \tau = t - \int_{z_0}^z (1/v_{g1}^{(s)}) dz'. \quad (32)$$

At the boundary, the probe beam is converted into a dark-state polariton $\Phi_{D1}^{(s)}(t)$ propagating at the group velocity $v_{g1}^{(s)} \ll c$ in the medium. Since the atomic population is created exclusively by the incident probe light, only the first dark-state polariton is populated, giving

$$\Phi_{D1}^{(s)} = \mathcal{E}^{(s)}/\zeta_c^{(s)}, \quad \Phi_{D2}^{(s)} = 0, \quad (33)$$

where the temporal and spatial dependences of the first polariton are kept implicit. In writing the last relationship, use has been made of Eq. (18) relating \mathcal{E} to Φ_{D1} and Φ_B , together with the adiabatic approximation implying that $\Phi_B \approx 0$. For slow light, the parameter $\zeta_c^{(s)} \approx \sqrt{v_{g1}^{(s)}/c}$ [featured in Eq. (33)] is much smaller than the unity. This is why the dark-state polariton $\Phi_{D1}^{(s)}$ contains only a tiny contribution by the electric field.

Equations (16) and (17) together with the condition $\Phi_B^{(s)}(t) = \Phi_{D2}^{(s)}(t) = 0$ provide the atomic fields (atomic coherences) associated with the first polariton:

$$\Phi_2^{(s)} = -\xi_{c2}^{(s)*} \zeta_1^{(s)} \Phi_{D1}^{(s)}, \quad \Phi_3^{(s)} = -\xi_{c3}^{(s)*} \zeta_1^{(s)} \Phi_{D1}^{(s)}. \quad (34)$$

At a certain time $t = t^{(s)}$, the whole probe pulse enters the atomic medium and is contained in it. To store the slow polariton, both control fields are switched off at $t = t^{(s)}$ in such a way that the Rabi frequency ratio $\Omega_{c2}^{(s)}/\Omega_{c3}^{(s)} = \xi_{c2}^{(s)}/\xi_{c3}^{(s)}$ remains constant, whereas $\zeta_1^{(s)} \rightarrow 1$. This gives the following atomic fields (atomic coherences) at the storing time:

$$\Phi_2^{(s)}(t^{(s)}) \rightarrow -\xi_{c2}^{(s)*} \Phi_{D1}^{(s)}(t^{(s)}), \quad \Phi_3^{(s)}(t^{(s)}) \rightarrow -\xi_{c3}^{(s)*} \Phi_{D1}^{(s)}(t^{(s)}). \quad (35)$$

The stored atomic coherences no longer have the radiative group velocity and thus are trapped in the medium. The retrieval of these coherences is accomplished at a later time $t = t^{(r)}$.

B. Regeneration of slow light

To restore the polariton propagation, both control fields are switched on again at $t = t^{(r)}$ in such a way that their the ratio $\Omega_{c2}^{(r)}/\Omega_{c3}^{(r)} = \xi_{c2}^{(r)}/\xi_{c3}^{(r)}$ is constant. The difference between the storage and the retrieval times should not be too large, so that

the atomic coherences given by Eq. (35) are preserved up to the retrieval time. In the initial experiment [7], the light was stored up to a millisecond, yet the storage duration was increased up to a second recently [16,17].

If the relative Rabi frequencies $\xi_{c2}^{(r)}$ and $\xi_{c3}^{(r)}$ differ from the original ones $\xi_{c2}^{(s)}$ and $\xi_{c3}^{(s)}$, both dark-state polaritons are regenerated. Using Eqs. (14) and (15), the dark-state polaritons regenerated from the atomic coherences (35), at the beginning of the release of light where $\zeta_1^{(r)} \approx 1$, read as

$$\Phi_{D1}^{(r)}(t^{(r)}) = (\xi_{c2}^{(r)} \xi_{c2}^{(s)*} + \xi_{c3}^{(r)} \xi_{c3}^{(s)*}) \Phi_{D1}^{(s)}(t^{(s)}), \quad (36)$$

$$\Phi_{D2}^{(r)}(t^{(r)}) = -(\xi_{c3}^{(r)*} \xi_{c2}^{(s)*} - \xi_{c2}^{(r)*} \xi_{c3}^{(s)*}) \Phi_{D1}^{(s)}(t^{(s)}). \quad (37)$$

The electric probe field reappears due to the first dark-state polariton containing a nonzero electric field contribution

$$\mathcal{E}^{(r)}(t) = \zeta_c^{(r)}(t) \Phi_{D1}^{(r)}(t). \quad (38)$$

Substitution of Eq. (36) into Eq. (38) and using Eq. (33), one can relate the regenerated electric field to the initial one as

$$\mathcal{E}^{(r)}(t^{(r)}) = \frac{\zeta_c^{(r)}}{\zeta_c^{(s)}} (\xi_{c2}^{(r)} \xi_{c2}^{(s)*} + \xi_{c3}^{(r)} \xi_{c3}^{(s)*}) \mathcal{E}^{(s)}(t^{(s)}). \quad (39)$$

If both the storing and the retrieval take place in the slow light regime $\Omega_c^{(s)} \ll g\sqrt{n}$ and $\Omega_c^{(r)} \ll g\sqrt{n}$, the above equation simplifies to

$$\mathcal{E}^{(r)}(t^{(r)}) = \frac{\Omega_{c2}^{(r)} \Omega_{c2}^{(s)*} + \Omega_{c3}^{(r)} \Omega_{c3}^{(s)*}}{|\Omega_{c2}^{(s)}|^2 + |\Omega_{c3}^{(s)}|^2} \mathcal{E}^{(s)}(t^{(s)}). \quad (40)$$

Propagation of the regenerated polariton $\Phi_{D1}^{(r)}$ is governed by Eq. (30) in the paraxial case. The polariton $\Phi_{D1}^{(r)}$ propagates at the velocity $v_{g1} \equiv v_{g1}^{(r)}$ and might experience diffraction effects due to the second-order transverse derivatives featured in Eq. (30). On the other hand, the second polariton Φ_{D2} is not coupled to the light fields and hence remains trapped (frozen) in the medium.

V. STORAGE AND RETRIEVAL OF SLOW LIGHT: SPECIFIC SITUATIONS

A. Restored control beams with the same spatial behavior

Let us first analyze the simplest situation where the Rabi frequencies of the restored control beams are proportional to the corresponding original ones with the same proportionality constant b :

$$\Omega_{c2}^{(r)} = b\Omega_{c2}^{(s)}, \quad \Omega_{c3}^{(r)} = b\Omega_{c3}^{(s)} \quad (41)$$

and hence $\xi_{c2}^{(r)} = \xi_{c2}^{(s)}$ and $\xi_{c3}^{(r)} = \xi_{c3}^{(s)}$. Under these conditions, Eqs. (36) and (37) together with (10) provide the following amplitudes of the regenerated dark-state polaritons:

$$\Phi_{D1}^{(r)}(t^{(r)}) = \Phi_{D1}^{(s)}(t^{(s)}), \quad \Phi_{D2}^{(r)} = 0. \quad (42)$$

Thus, the second polariton is not populated ($\Phi_{D2}^{(r)} = 0$), whereas the first regenerated dark-state polariton coincides with the original one. The corresponding regenerated electric field

$$\mathcal{E}^{(r)} = b\mathcal{E}^{(s)} \quad (43)$$

is proportional to the original one and thus does not acquire the phase singularity of the control beam Ω_{c2} (if any). In such a situation, the vortex can not be transferred from the control to the regenerated probe beam. In the following subsections, we will analyze the vortex transfer from the control beam Ω_{c2} to the regenerated probe beam in the case for which the condition (41) no longer holds. Such a vortex transfer is accompanied with some population of the second polariton.

It is noteworthy that the regenerated electric field $\mathcal{E}^{(r)}$ given by Eq. (43) is increased (decreased) if the ratio of the total Rabi frequencies $b = \Omega_c^{(r)}/\Omega_c^{(s)}$ is larger (smaller) than the unity. On the other hand, the group velocity is increased for $b > 1$ and decreased for $b < 1$. This leads to the compression (for $b < 1$) or decompression for ($b > 1$) of the regenerated probe pulse as compared to the stored one, a feature known from the light storage and retrieval in the Λ system [7]. Note also that the total number of the regenerated photons is the same as that in the input beam. This is because the second polariton is not populated $\Phi_{D2} = 0$, so no atomic coherence remains frozen in the medium.

B. Transfer of optical vortex at the retrieval of the probe beam

Suppose that only one control field is used during the storage phase of the probe light, i.e., $\Omega_{c3}^{(s)} = 0$ and hence $|\xi_{c2}^{(s)}| = 1$. This means that the storage stage involves a Λ scheme as depicted in Fig. 1. In such a setup, the control beam $\Omega_{c2}^{(s)}$ can not carry an OAM; otherwise, there would be nonadiabatic losses of the probe beam at the vortex core of the control beam. On the other hand, the retrieval of the probe beam is accomplished using a tripod system in which generally both $\Omega_{c2}^{(r)}$ and $\Omega_{c3}^{(r)}$ are nonzero. Under these conditions, Eqs. (36) and (37) provide the following results for the regenerated polaritons:

$$\Phi_{D1}^{(r)}(t_i^{(r)}) = \xi_{c2}^{(r)} \xi_{c2}^{(s)*} \Phi_{D1}^{(s)}(t_f^{(s)}), \quad (44)$$

$$\Phi_{D2}^{(r)}(t_i^{(r)}) = -\xi_{c3}^{(r)*} \xi_{c2}^{(s)*} \Phi_{D1}^{(s)}(t_f^{(s)}). \quad (45)$$

The second polariton given by Eq. (45) does not have a radiative component and is trapped in the medium. The electric field of the probe beam is regenerated exclusively due to the first polariton and reads as [using Eq. (40)]

$$\mathcal{E}^{(r)} = \frac{\Omega_{c2}^{(r)}}{\Omega_{c2}^{(s)}} \mathcal{E}^{(s)}(t^{(s)}). \quad (46)$$

Equations (44) or (46) represent the initial condition for the subsequent propagation of the polariton in the medium. Such a polariton will propagate along the z axis with the group velocity v_{rad} . Its transverse profile will change due to the diffraction effects represented by the second-order spatial derivatives in Eq. (30).

If the control beam Ω_{c2} carries an optical vortex at the retrieval stage $\Omega_{c2}^{(r)} \sim e^{i\ell\varphi}$, the regenerated electric field $\mathcal{E}^{(r)} \sim e^{i\ell\varphi}$ acquires the same phase as one can see from Eq. (46). This means that the restored control beam transfers its optical vortex to the regenerated electric field $\mathcal{E}^{(r)}$. If the initial control

and probe fields have the same transverse dependence, the transverse profile of the regenerated electric field will mimic that of the control field $\mathcal{E}^{(r)} \sim \Omega_{c2}^{(r)} \sim e^{i\ell\varphi}$.

As an illustration, let us take the restoring control laser $\Omega_{c2}^{(r)}$ to be the first-order Laguerre-Gaussian (LG) beam $\Omega_{c2}^{(r)} = A \tilde{\rho} e^{i\varphi} \exp(-\tilde{\rho}^2/\sigma_r^2)$, where $\tilde{\rho} = \rho/\lambda$ is a dimensionless cylindrical radius, with $\lambda = 2\pi/k$ being the optical wavelength. On the other hand, the control beam is assumed to be the zero-order LG beam during the storage stage involving a Λ system: $\Omega_{c2}^{(s)} = a^{-1} A \exp(-\tilde{\rho}^2/\sigma_s^2)$, where a determines a relative amplitude of the control fields $\Omega_{c2}^{(r)}$ and $\Omega_{c2}^{(s)}$, with σ_r and σ_s being their dimensionless widths. This provides the regenerated probe field

$$\mathcal{E}^{(r)} = a \tilde{\rho} e^{i\varphi} \exp[-\tilde{\rho}^2(\sigma_r^{-2} - \sigma_s^{-2})] \mathcal{E}^{(s)}(t^{(s)}). \quad (47)$$

It is noteworthy that the Rabi frequency of the additional laser $\Omega_{c3}^{(r)}$ does not enter the above equations (46) and (47) for the regenerated probe field. Yet, the additional laser plays an important role to ensure that the lossless propagation of the restored probe field in a vicinity of the vortex core where $\Omega_{c2}^{(r)} \rightarrow 0$, as one can see from Eq. (31).

C. Transfer of the optical vortex during the storage of slow light

Consider next the opposite situation in which both control fields are on during the storage phase, so the storage of the probe beam is carried out using a tripod scheme. On the other hand, a Λ scheme is employed during the retrieval of the probe beam where only one control field is on, i.e., $\Omega_{c3}^{(r)} = 0$ and hence $|\xi_{c2}^{(r)}| = 1$. In this case, Eqs. (36) and (37) yield the following results for the regenerated polaritons:

$$\Phi_{D1}^{(r)}(t_i^{(r)}) = \xi_{c2}^{(r)} \xi_{c2}^{(s)*} \Phi_{D1}^{(s)}(t_f^{(s)}), \quad (48)$$

$$\Phi_{D2}^{(r)}(t_i^{(r)}) = \xi_{c2}^{(r)*} \xi_{c3}^{(s)*} \Phi_{D1}^{(s)}(t_f^{(s)}). \quad (49)$$

Again, the electric probe field is regenerated exclusively due to the first polariton and is given by [using Eq. (40)]

$$\mathcal{E}^{(r)} = \frac{\Omega_{c2}^{(r)} \Omega_{c2}^{(s)*}}{|\Omega_{c2}^{(s)}|^2 + |\Omega_{c3}^{(s)}|^2} \mathcal{E}^{(s)}(t^{(s)}). \quad (50)$$

Equations (48) or (50) represent the initial conditions for the subsequent propagation of the regenerated polariton governed, in the paraxial case, by the equation of motion (30). Such a polariton will propagate along the z axis with the group velocity v_{rad} , and its transverse profile will change due to the diffraction effects represented by the second-order spatial derivatives in Eq. (30). On the other hand, the second polariton Φ_{D2} will be frozen in the medium (neglecting the atomic motion), and its spatial form is given by Eq. (49).

If the second control beam carries an optical vortex at the storing stage $\Omega_{c2}^{(s)} \sim e^{i\ell\varphi}$, the regenerated electric field $\mathcal{E} \sim e^{-i\ell\varphi}$ acquires an opposite vorticity, as one can see from Eqs. (48) and (50). The additional control beam $\Omega_{c3}^{(s)}$ does not have a vortex and hence is nonzero at the center. This ensures the lossless (adiabatic) propagation of the probe beam during the storage phase. It is noteworthy that the transverse profile

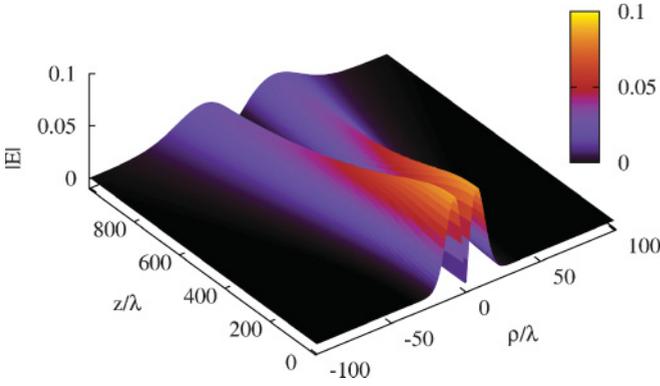


FIG. 3. (Color online) Propagation of the regenerated probe field in the free space for $\sigma = 10$ and $a = 1$. The Λ scheme is used for storage and the tripod system is used for retrieval of the probe field.

of the regenerated probe field differs now from that of the storing beam $\Omega_{c2}^{(s)}$ because of the denominator in Eq. (50).

Suppose that the control lasers are the first- and zero-order LG beams at the storage stage

$$\Omega_{c2}^{(s)} = A\tilde{\rho}e^{i\varphi} \exp(-\tilde{\rho}^2/\sigma_s^2), \quad \Omega_{c3}^{(s)} = bA \exp(-\tilde{\rho}^2/\sigma_s^2), \quad (51)$$

where the parameter b determines the relative amplitude of the additional control laser. On the other hand, the control beam is assumed to be the zero-order LG beam at the retrieval stage involving the Λ scheme: $\Omega_{c2}^{(r)} = aA \exp(-\tilde{\rho}^2/\sigma_r^2)$. Thus, one arrives at the following regenerated probe field containing the phase-conjugated vortex:

$$\mathcal{E}^{(r)} = \frac{a}{\tilde{\rho}^2 + b^2} \tilde{\rho} e^{-i\varphi} \exp[-\tilde{\rho}^2(\sigma_r^{-2} - \sigma_s^{-2})] \mathcal{E}^{(s)}(t^{(s)}). \quad (52)$$

It is noteworthy that for $b < 1$, the transverse profile of the regenerated beam can differ considerably from the the Laguerre-Gaussian shape. By decreasing b , the transverse shape of the regenerated beam becomes narrower. This leads to a larger diffraction in its subsequent propagation, as will be explored in the following subsection.

D. Dynamics of the restored probe beams

Let us suppose that the atomic cloud is small enough so that the diffraction can be neglected during the propagation

of the probe beam in the medium. Such a condition can be fulfilled readily for a typical cloud of cold atoms, the length of which normally does not exceed a third of a millimeter [1]. After leaving the atomic cloud, the probe beam propagates in the free space according to Eq. (2) with $g = 0$. Since the probe field is quasimonochromatic, its amplitude $\mathcal{E}(\mathbf{r}, t)$ changes little during an optical cycle. In the stationary case, one arrives at the following propagation equation for the slowly varying amplitude of the probe field:

$$i \frac{\partial}{\partial z} \mathcal{E} = -\frac{1}{2k} \nabla_{\perp}^2 \mathcal{E}. \quad (53)$$

In the previous two subsections, we have considered two possible scenarios to regenerate the probe field. In the first (Λ - T) case, the lambda scheme is used for storing the probe field, whereas the tripod setup is employed for the regeneration. In the second (T - Λ) case, the tripod scheme is used for storing the probe field, whereas the lambda setup is used for the regeneration. In what follows, we shall explore the subsequent propagation of the probe beam. The regenerated fields given by Eqs. (47) and (52) represent the initial conditions for such a propagation. By taking the initial probe beam to be Gaussian $\mathcal{E}^{(s)} = \mathcal{E}_0^{(s)} \exp(-\tilde{\rho}^2/\sigma_p^2)$, the regenerated fields for both cases read as

$$\begin{aligned} \mathcal{E}_{\Lambda-T}^{(r)} &= a\mathcal{E}_0^{(s)} \tilde{\rho} e^{i\varphi} e^{-\tilde{\rho}^2/\sigma^2}, \\ \mathcal{E}_{T-\Lambda}^{(r)} &= \frac{a}{\tilde{\rho}^2 + b^2} \mathcal{E}_0^{(s)} \tilde{\rho} e^{-i\varphi} e^{-\tilde{\rho}^2/\sigma^2}, \end{aligned} \quad (54)$$

where $\sigma^{-2} = \sigma_p^{-2} + \sigma_r^{-2} - \sigma_s^{-2}$ determines the width of the regenerated probe field measured in optical wavelength $\lambda = 2\pi/k$.

Equation (53) has been solved numerically for $\sigma = 10$. Figure 3 shows the subsequent propagation of the regenerated beam for the first case. Here the regenerated field $\mathcal{E}_{\Lambda-T}^{(r)}$ represents the first-order LG beam and is proportional to the relative intensity of the control field at the release and storage stages $\Omega_{c2}^{(r)}/\Omega_{c2}^{(s)}$. The subsequent propagation of the field qualitatively preserves the transverse profile and is accompanied by some of the diffraction spreading.

Figures 4(a)–4(c) illustrate the second case in which the tripod scheme is used for the storing and the Λ scheme is used for the retrieval of the probe beam. The transverse profile of the regenerated beam $\mathcal{E}_{T-\Lambda}^{(r)}$ is determined by the relative intensity b of the additional laser beam $\Omega_{c3}^{(r)}$. For $b = 3$, the profile is much narrower as compared to the first-order LG beam with

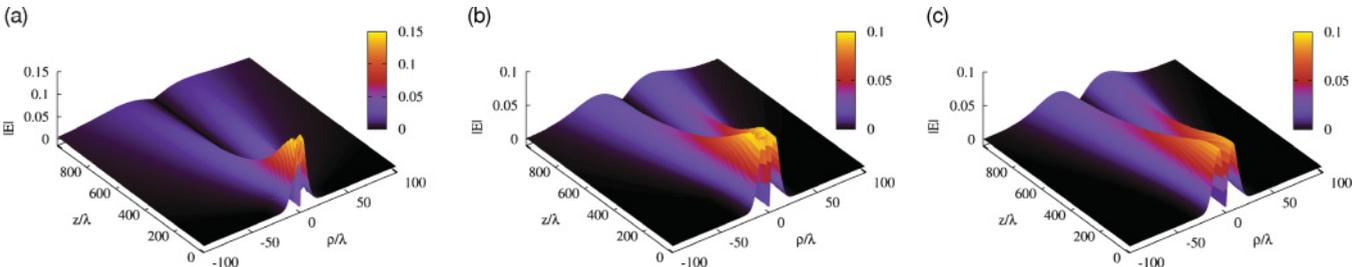


FIG. 4. (Color online) Propagation of the regenerated electric field in the free space in the case of a tripod system for storage and a Λ system for retrieval by taking $\sigma = 10$ and $a = 1$. The parameter b appearing in Eq. (54) is $b = 3, 10,$ and 30 in (a), (b), and (c), respectively.

the same width σ , as one can see comparing Figs. 3 and 4(a). Consequently, the light beam spreads out much faster than that in the first case [see Fig. 4(a)]. When b increases [$b = 10$ in Fig. 4(b) and $b = 30$ in Fig. 4(c)], the profile of the probe beam approaches the shape featured in the first case. Note that the increase in the relative intensity b of the additional control laser is accompanied by the decrease in the intensity of the regenerated probe beam. Thus, the improvement in the quality of the regenerated beams comes at the cost of reducing its intensity.

VI. CONCLUDING REMARKS

Polariton formalism has been applied for studying the propagation of a probe field of light in a cloud of cold atoms influenced by two additional control laser beams of larger intensity. The probe and control beams couple resonantly three hyperfine ground states to a common excited state in a tripod configuration of the atomic energy levels. The first control beam can have an optical vortex. Application of another control beam without a vortex ensures the lossless (adiabatic) propagation of the probe beam at the vortex core where the intensity of the first control beam goes to zero. The adiabatic propagation of the probe beam is obtained when the total intensity of the control lasers is sufficiently large at the vortex core.

We have started with a set of atomic equations coupled with the equation for the probe beam, subsequently transforming them into two coupled equations for the dark-state polaritons. We have analyzed conditions (related to the laser pulse durations and switching times) when the polaritons are decoupled and thus the problem reduces to a single equation for the polariton. An advantage of polaritonic analysis is the simplicity of the relationship between the polariton field and the regenerated electric field. Furthermore, the equation for the polariton has a usual form of matter wave equation, which also describes the atomic evolution when the control fields are off.

The probe pulse is stored onto the atomic coherences and subsequently retrieved by switching off and on the control beams. As a result, the optical vortex can be transferred from the control to the probe fields during the storage or retrieval. Two scenarios have been analyzed in more detail. The first case involves a Λ system for the storage and a tripod system for the retrieval. In such a situation, the phase of the vortex is transferred from the restoring control beam to the regenerated probe beam. In the second case, the tripod system is used for the storage and the Λ system is used for the retrieval. The vortex phase is then transferred from the storing control beam to the regenerated probe beam in the phase-conjugated form, so the probe beam acquires an opposite vorticity. The profile of the regenerated probe field is well preserved in the first case. On the other hand, in the second case, the regenerated beam becomes narrower and thus experiences larger diffraction spreading. The width of the regenerated beam can be controlled by changing the intensity of the additional control beam during the storage phase.

The tripod setup can be realized for atoms such as sodium [7] or rubidium [8] containing two hyperfine ground levels with $F = 1$ and 2, as depicted in Fig. 2(c). The scheme can be produced by adding an extra circularly polarized laser

beam Ω_{c3} as compared to the experiment by Liu *et al.* [7] on the light storage in the gases using the Λ scheme. Thus, it is feasible to implement the suggested experiment on the transfer of the optical vortex from the control to the probe fields using the tripod setup.

ACKNOWLEDGMENT

The authors acknowledge the support by the Research Council of Lithuania (Grants No. MOS-2/2010 and No. VP1-3.1-MM-01-V-01-001) and the EU FP7 project STREP NAMEQUAM.

APPENDIX A: MATRIX ELEMENTS IN THE EQUATION FOR THE DARK POLARITONS

The elements of the matrix \mathbf{J} featured in Eq. (19) are

$$\mathbf{J}_{11} = i\hbar \left(\frac{c^2}{\hbar\omega} \zeta_c \nabla \zeta_c + \frac{1}{m} \zeta_1^* \nabla \zeta_1 \right) + |\zeta_1|^2 \mathbf{J}_{B2}, \quad (\text{A1})$$

$$\mathbf{J}_{22} = i \frac{\hbar}{m} (\xi_{c3}^* \nabla \xi_{c3} + \xi_{c2}^* \nabla \xi_{c2}), \quad (\text{A2})$$

$$\mathbf{J}_{12} = i \frac{\hbar}{m} \zeta_1^* (\xi_{c3} \nabla \xi_{c2} - \xi_{c2} \nabla \xi_{c3}), \quad (\text{A3})$$

where

$$\mathbf{J}_{B2} = i \frac{\hbar}{m} (\xi_{c2} \nabla \xi_{c2}^* + \xi_{c3} \nabla \xi_{c3}^*). \quad (\text{A4})$$

The elements of the matrix U in Eq. (19) read as

$$U_{11} = -\frac{\hbar^2}{2} \left(\frac{c^2}{\hbar\omega} \zeta_c \nabla^2 \zeta_c + \frac{1}{m} \zeta_1^* \nabla^2 \zeta_1 \right) + i\hbar \zeta_1^* \nabla \zeta_1 \cdot \mathbf{J}_{B2} + |\zeta_1|^2 U_{B2} - \frac{\hbar\omega}{2} \zeta_c^2 + i\hbar \left(\zeta_c \frac{\partial}{\partial t} \zeta_c + \zeta_1 \frac{\partial}{\partial t} \zeta_1^* \right), \quad (\text{A5})$$

$$U_{22} = -\frac{\hbar^2}{2m} (\xi_{c3}^* \nabla^2 \xi_{c3} + \xi_{c2}^* \nabla^2 \xi_{c2}) + [\hbar\omega_{21} + V_2(\mathbf{r})] |\xi_{c3}|^2 + [\hbar\omega_{31} + V_3(\mathbf{r})] |\xi_{c2}|^2 + i\hbar \left(\xi_{c3} \frac{\partial}{\partial t} \xi_{c3}^* + \xi_{c2} \frac{\partial}{\partial t} \xi_{c2}^* \right), \quad (\text{A6})$$

$$U_{12} = -\frac{\hbar^2}{2m} \zeta_1^* (\xi_{c3} \nabla^2 \xi_{c2} - \xi_{c2} \nabla^2 \xi_{c3}) + \zeta_1^* \xi_{c2} \xi_{c3} [\hbar\omega_{32} + V_3(\mathbf{r}) - V_2(\mathbf{r})] + i\hbar \zeta_1^* \left(\xi_{c2} \frac{\partial}{\partial t} \xi_{c3} - \xi_{c3} \frac{\partial}{\partial t} \xi_{c2} \right), \quad (\text{A7})$$

$$U_{21} = -\frac{\hbar^2}{2m} \zeta_1 (\xi_{c2}^* \nabla^2 \xi_{c3}^* - \xi_{c3}^* \nabla^2 \xi_{c2}^*) + i\hbar \frac{1}{\zeta_1} \nabla \zeta_1 \cdot \mathbf{J}_{21} + \zeta_1 \xi_{c2}^* \xi_{c3}^* [\hbar\omega_{32} + V_3(\mathbf{r}) - V_2(\mathbf{r})] + i\hbar \zeta_1 \left(\xi_{c3}^* \frac{\partial}{\partial t} \xi_{c2}^* - \xi_{c2}^* \frac{\partial}{\partial t} \xi_{c3}^* \right), \quad (\text{A8})$$

where

$$U_{B2} = -\frac{\hbar^2}{2m} (\xi_{c2} \nabla^2 \xi_{c2}^* + \xi_{c3} \nabla^2 \xi_{c3}^*) + [\hbar\omega_{21} + V_2(\mathbf{r})] |\xi_{c3}|^2 + [\hbar\omega_{31} + V_3(\mathbf{r})] |\xi_{c2}|^2 + i\hbar \left(\xi_{c2} \frac{\partial}{\partial t} \xi_{c2}^* + \xi_{c3} \frac{\partial}{\partial t} \xi_{c3}^* \right). \quad (\text{A9})$$

APPENDIX B: MATRIX ELEMENTS IN THE PARAXIAL EQUATION FOR THE DARK POLARITONS

Using Eq. (24), the parameters ξ_{c2} and ξ_{c3} have the form $\xi_{c2} = \xi'_{c2} e^{ik_c z}$, $\xi_{c3} = \xi'_{c3} e^{ik_c z}$, where ξ'_{c2} and ξ'_{c3} slowly change with the distance z within the optical wavelength. The diagonal elements of the matrix \mathbf{J}' entering Eq. (27) are given by

$$\mathbf{J}'_{11} = i\hbar \left(\frac{c^2}{\hbar\omega} \zeta_c \nabla \zeta_c + \frac{1}{m} \zeta_1^* \nabla \zeta_1 \right) + |\zeta_1|^2 \mathbf{J}'_{B2}, \quad (\text{B1})$$

$$\mathbf{J}'_{22} = i \frac{\hbar}{m} (\xi_{c3}^* \nabla \xi'_{c3} + \xi_{c2}^* \nabla \xi'_{c2}), \quad (\text{B2})$$

where

$$\mathbf{J}'_{B2} = i \frac{\hbar}{m} (\xi'_{c2} \nabla \xi_{c2}^* + \xi'_{c3} \nabla \xi_{c3}^*). \quad (\text{B3})$$

The nondiagonal matrix elements of \mathbf{J}' read as

$$\mathbf{J}'_{12} = \mathbf{J}_{12} e^{-i(k+k_c)z} = i \frac{\hbar}{m} \zeta_1^* e^{i(k_c-k)z} (\xi_{c3}' \nabla \xi'_{c2} - \xi_{c2}' \nabla \xi'_{c3}), \quad (\text{B4})$$

$$\mathbf{J}'_{21} = \mathbf{J}_{21} e^{i(k+k_c)z} = \mathbf{J}_{12}^*. \quad (\text{B5})$$

The diagonal matrix elements of the complex scalar potential U' in Eq. (27) are

$$U'_{11} = -\frac{\hbar^2}{2} \left(\frac{c^2}{\hbar\omega} \zeta_c \nabla^2 \zeta_c + \frac{1}{m} \zeta_1^* \nabla^2 \zeta_1 \right) + i\hbar \zeta_1^* \nabla \zeta_1 \cdot \mathbf{J}'_{B2} + |\zeta_1|^2 \left(U'_{B2} + \frac{\hbar^2(k-k_c)^2}{2m} \right) + i\hbar \left(\zeta_c \frac{\partial}{\partial t} \zeta_c + \zeta_1 \frac{\partial}{\partial t} \zeta_1^* \right), \quad (\text{B6})$$

$$U'_{22} = -\frac{\hbar^2}{2m} (\xi_{c3}^* \nabla^2 \xi'_{c3} + \xi_{c2}^* \nabla^2 \xi'_{c2}) + [\hbar\omega_{21} + V_2(\mathbf{r})] |\xi_{c3}|^2 + [\hbar\omega_{31} + V_3(\mathbf{r})] |\xi_{c2}|^2 + i\hbar \left(\xi'_{c3} \frac{\partial}{\partial t} \xi_{c3}^* + \xi'_{c2} \frac{\partial}{\partial t} \xi_{c2}^* \right), \quad (\text{B7})$$

where

$$U'_{B2} = -\frac{\hbar^2}{2m} (\xi'_{c2} \nabla^2 \xi_{c2}^* + \xi'_{c3} \nabla^2 \xi_{c3}^*) + [\hbar\omega_{21} + V_2(\mathbf{r})] |\xi_{c2}|^2 + [\hbar\omega_{31} + V_3(\mathbf{r})] |\xi_{c3}|^2 + i\hbar \left(\xi_{c2}^* \frac{\partial}{\partial t} \xi'_{c2} + \xi_{c3}^* \frac{\partial}{\partial t} \xi'_{c3} \right). \quad (\text{B8})$$

Finally, the nondiagonal elements of the complex scalar potential U' are given by

$$U'_{12} = -\frac{\hbar^2}{2m} \zeta_1^* e^{i(k_c-k)z} (\xi_{c3}' \nabla^2 \xi'_{c2} - \xi_{c2}' \nabla^2 \xi'_{c3}) + \zeta_1^* e^{i(k_c-k)z} \zeta_{c2}' \zeta_{c3}' [\hbar\omega_{32} + V_3(\mathbf{r}) - V_2(\mathbf{r})] + i\hbar \zeta_1^* e^{i(k_c-k)z} \left(\xi_{c2}' \frac{\partial}{\partial t} \xi'_{c3} - \xi_{c3}' \frac{\partial}{\partial t} \xi'_{c2} \right),$$

$$U'_{21} = -\frac{\hbar^2}{2m} \zeta_1 e^{i(k-k_c)z} (\xi_{c2}^* \nabla^2 \xi_{c3}^* - \xi_{c3}^* \nabla^2 \xi_{c2}^*) + i\hbar \frac{1}{\zeta_1} \nabla \zeta_1 \cdot \mathbf{J}'_{21} + \zeta_1 e^{i(k-k_c)z} \zeta_{c2}^* \zeta_{c3}^* [\hbar\omega_{32} + V_3(\mathbf{r}) - V_2(\mathbf{r})] + i\hbar \zeta_1 e^{i(k-k_c)z} \left(\xi_{c3}^* \frac{\partial}{\partial t} \xi_{c2}^* - \xi_{c2}^* \frac{\partial}{\partial t} \xi_{c3}^* \right).$$

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