Effective Magnetic Fields for Stationary Light

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We describe a method to create effective gauge potentials for stationary-light polaritons. When stationary light is created in the interaction with a rotating ensemble of coherently driven double-\(\Lambda\) polaritons, the equation of motion is that of a massive Schrödinger particle in a magnetic field. Since the effective interaction area for the polaritons can be made large, degenerate Landau levels can be created with degeneracy well above 100. This opens up the possibility to study the bosonic analogue of the fractional quantum Hall effect for interacting stationary-light polaritons.

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One of the outstanding and challenging problems of many-body physics is the understanding of strongly correlated quantum systems. With the advances of atomic physics and quantum optics over the last decades a number of new model systems based on cold atoms emerged which allow an experimental study with unprecedented precision and control [1]. Recently it has been suggested to consider quasiparticles of light-matter interaction as an alternative. It has been predicted in [2] that slow-light polaritons [3] in a nonlinear fiber would undergo a crystallization similar to the Tonks-Girardeau transition [4]. The dynamics of interacting polaritons was considered in a cavity array [5], realizing the Jaynes-Cummings-Hubbard model. Furthermore a mechanism to induce Bose-Einstein condensation [6] of stationary-light polaritons [7] was proposed and analyzed.

In this Letter we show that it is possible to create effective magnetic fields for stationary-light polaritons. This extends previous proposals for the generation of effective magnetic fields for stationary-light polaritons. In the following we show how a nonzero effective magnetic field can be generated for stationary light using a uniformly rotating medium, similar to cold atoms in rotating traps [1,15]. In the case of electrically neutral cold atoms an artificial magnetic field can also be created using two counter-propagating light beams with shifted spatial profiles [16,17] or having a transverse dependence of the atomic energy levels [18]. Yet in the present scheme it is the stationary polaritons rather than the atoms that are affected by the gauge field. The underlying mechanism can be attributed to a rotational frequency shift [19,20]. There have been proposals of creating gauge fields for slow-light via the spatial dependence of the control beams [21] or using moving media to induce an Aharonov-Bohm phase [22] and light drag [23,24] for slow light. In contrast, in the present scheme it is the stationary polaritons rather than the atoms that are affected by the gauge field. The underlying mechanism can be attributed to a rotational frequency shift [19,20].

We consider a four level scheme involving two hyperfine atomic ground states \(|g\rangle\) and \(|i\rangle\) with magnetic quantum numbers \(m = 0\), as well as two excited states \(|e_\pm\rangle\) with \(m = \pm 1\), as shown in Fig. 1. The states are coupled in a cavity array [5], realizing the Jaynes-Cummings-Hubbard model. Furthermore a mechanism to induce Bose-Einstein condensation [6] was proposed and analyzed.

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FIG. 1 (color online). The Raman interaction of two counter-propagating control lasers of Rabi frequencies \(\Omega_{\pm} e^{i\kappa z}\) drives the transi-
Here we put \(|s| \to |e_z|\) to create electromagnetically induced transparency (EIT) for another pair of counterpropagating quantized probe fields \(\hat{E}_s\) coupling the states \(|g\rangle\) and \(|e_z\rangle\). This sets up two parallel \(\Lambda\) schemes sharing the same ground states. EIT appears for two-photon resonance in both \(\Lambda\) systems \(\omega_+^{(p)} - \omega_+^{(c)} = \omega_+^{(p)} - \omega_+^{(c)} = \omega_{\text{sc}}\). If the amplitudes of the control fields are equal, \(|\Omega_+\rangle = |\Omega_-\rangle\), a stationary-light polariton is formed [6,7].

Let us introduce field amplitudes \(\hat{E}_s\) that are normalized to a number and vary slowly in space and time by \(\hat{E}_s(r, t) = \sqrt{\frac{\hbar}{2\varepsilon_0 \varepsilon}} \hat{E}_s(r, t) \exp(-i(\omega_{pt} + kpz)) + \text{H.c.}\)

Furthermore continuous atomic-flip operators are defined as \(\hat{\sigma}_{\mu \nu}(r, t) = \frac{1}{\Delta N} \sum_{\nu \in \Delta N} \hat{\sigma}_{\mu \nu}\), where \(\hat{\sigma}_{\mu \nu} \equiv |\mu \rangle \langle \nu|\) is the flip operator of the \(j\)th atom, and the sum is taken over a small volume \(\Delta V\) around \(r\) containing \(\Delta N\) atoms.

In what follows the probe fields are considered weak. This prevents depletion of the ground state \(|g\rangle\). We further assume the medium to rotate uniformly with angular frequency \(\nu\) around the propagating direction of control and probe fields. As a result one arrives at a set of equations for the atomic coherences and the probe beams in the lab frame:

\[
i \frac{\partial}{\partial t} \hat{\sigma}_{gs} = \delta \hat{\sigma}_{gs} - \Omega_+ \hat{\sigma}_{ge -} - \Omega_- \hat{\sigma}_{ge +},
\]

\[
i \frac{\partial}{\partial t} \hat{\sigma}_{ge +} = -i \Gamma_\pm \hat{\sigma}_{ge +} - \Omega_\pm \hat{\sigma}_{gs} - g \sqrt{n} \hat{E}_s + i \hat{F}_s,
\]

where \(\nu_s = \cos^2 \theta\) is the EIT group velocity and \(\nabla_\perp = (\hat{\sigma}_x, \hat{\sigma}_y)^T\). Here \(\hat{\Phi}_1 = -\sin \varphi \hat{E}_+ + \cos \varphi \hat{E}_+\) and \(\hat{\Phi}_2 = \sin \varphi \hat{E}_+ + \cos \varphi \hat{E}_-\) are superpositions of the other eigensolutions of Eqs. (1)–(3) whose equations of motion after elimination of the excited states read

\[
\left[ \frac{\partial}{\partial t} - i \frac{c}{2k_p} \nabla_\perp^2 \right] \hat{\Phi}_2 = -i \varepsilon \sin \theta \cos \theta \hat{\Phi}_1 + \hat{F}_2,
\]

\[
\left[ \frac{\partial}{\partial t} - i \frac{c}{2k_p} \nabla_\perp^2 \right] \hat{\Phi}_2 = -i \varepsilon \sin \theta \cos \theta \hat{\Phi}_1 - \frac{\hbar^2}{m_{\perp}} \hat{\Phi}_2 + \hat{F}_2,
\]

Here we put \(\Gamma_\pm = \Gamma = \Gamma\) and neglected terms containing \(\delta\) because of the EIT condition \(\Gamma \delta \ll \Omega^2\). \(\hat{\Phi}_{1,2}\) are the Langevin noise forces of the bright polaritons. Adiabatic elimination of \(\hat{\Phi}_{1,2}\) in Eqs. (6) and (7) and subsequent substitution into Eq. (5) results in

\[
\sqrt{\frac{\hbar}{2m_{\parallel}} \hat{p}_z^2 + \frac{1}{2m_{\perp}} \left( \hat{\Phi}_1 + A \right)^2 + U} \hat{\Psi} - i \frac{\Gamma_{\parallel}}{\hbar} \hat{L}_z^2 \hat{\Psi} - i \hbar D_{\text{diff}}^{\parallel} \frac{\partial}{\partial z^2} \hat{\Psi} + \hat{F}_s,
\]

which is correct up to second order in nonadiabatic corrections. Equation (8) represents a Schrödinger equation for a particle with an effective tensorial mass and with minimal coupling to an artificial gauge field. The masses are given by
$m_{\perp} = \hbar \gamma / 2 v_g L_{\text{abs}} \Delta$ and $m_{\perp} = \hbar k_p / v_g$, respectively. 
$L_{\text{abs}} = c \gamma / g^2 n$ defines the resonant absorption length in absence of EIT and $F_\psi$ is the Langevin noise force for the dark-state polariton. The emerging gauge potential can be expressed as

$$\mathbf{A} = m_{\perp} (v_{\perp} \times \mathbf{r}_\perp) \sin^2 \theta,$$

where $\mathbf{r}_\perp = (x, y)^T$ is the radial vector from the axis and the corresponding momentum operator is given by $\mathbf{p}_\perp = -i \hbar \nabla_\perp$. The scalar potential reads

$$U = -\frac{1}{2} m_{\perp} v_{\perp}^2 \rho^2 \sin^4 \theta + h \rho \sin^2 \theta,$$

where $\rho = ||\mathbf{r}_\perp||$. Choosing a proper two-photon detuning, by, e.g., spatially varying Zeeman or Stark shifts, the antibinding centrifugal potential can be compensated. Finally the rotation induced and longitudinal diffusion rates $\Gamma_\text{rot}$ and $D_\text{diff}^\|$ are

$$\Gamma_\text{rot}^\perp = \frac{L_{\text{abs}}}{v_g} \frac{v_{\perp}^2 \sin^2 \theta \cos^4 \theta (1 + i \frac{\Delta}{\gamma})}{1},$$

$$D_\text{diff}^\| = v_{\perp} L_{\text{abs}}.$$

The real part of $\Gamma_\text{rot}^\perp$ describes azimuthal diffusion associated with loss and the imaginary part a corresponding correction to the mass. $D_\text{diff}^\|$ is responsible for a diffusive behavior along the original propagation axis of the light.

From Eq. (9) we compute the magnetic field to be

$$\mathbf{B} = \nabla \times \mathbf{A} = 2 m_{\perp} \nu \sin^2 \theta \mathbf{e}_z.$$

This expression is identical to the effective magnetic field $B_{\text{eff}} = 2 m_{\perp} \nu$ created by rotating cold gases [15] except for the factor $\sin^2 \theta$ and the substitution of atomic mass with the mass of the quasiparticles. This can be understood as follows: The particles feeling the magnetic field are the polaritons rather than the atoms. According to Eq. (4) these particles are a superposition of photonic and matter excitation and only the matter component, proportional to $\sin^2 \theta$ and with effective mass $m_{\perp}$, is subject to rotations. For $\sin^2 \theta = 0$ there is no coupling between light and medium and thus no gauge potential emerges for the polariton which in this case is just the electromagnetic field.

From Eq. (13) we obtain magnetic length and filling factor

$$L_{\text{mag}}^2 = \frac{\hbar}{B} = \frac{1}{4 \pi} \lambda R \frac{v_{\perp}}{v_{\text{rot}}},$$

$$\nu_{\text{filling}} = 2 \pi n \Psi L_{\text{mag}}^2 = \frac{1}{2} N \Psi \frac{\lambda}{R} \frac{v_{\perp}}{v_{\text{rot}}}. $$

Here we introduced the rotation velocity $v_{\text{rot}} = \nu R$ of the medium at its circumference $\rho_{\text{max}} = R, N \Psi$ is the number of DSPs and $\lambda$ is the wavelength of the probe field. Taking realistic values of $\lambda = 500 \text{ nm}$, $R = 5 \text{ mm}$, $\nu = 1 \text{ kHz}$ and values of $v_{\perp}$ between $10^3 \text{ m/s}$ down to $20 \text{ m/s}$ [25], yields a degeneracy of the lowest Landau level $R^2/(2 \pi L_{\text{mag}}^2)$ between 100 and $5 \times 10^3$.

The adiabaticity condition imposed by rate (11) reads $\operatorname{Re}[\Gamma_{\text{rot}}] \ll \omega_c$, where the cyclotron frequency $\omega_c = B/m_{\perp} = 2 \nu \sin^2 \theta$ sets the time scale of the relevant physics. Analogously we obtain a lower bound for the rotation frequency by the adiabaticity condition from rate (12) leading to $\omega_c \gg D_{\text{diff}}^\|/L_p^\|$. This yields the following conditions

$$\frac{1}{2} \frac{v_{\perp}}{L_{\text{abs}}} \left( L_{\text{abs}} / L_p \right)^2 \ll \nu \ll \frac{v_{\perp}}{L_{\text{abs}}} \frac{1}{\cos^4 \theta}. $$

Here $L_p$ stands for the characteristic length scale of the stationary DSP along the $z$ axis. The right side is easily fulfilled, since for a typical group velocity $v_g \sim 10^3 \text{ m/s}$ the resulting mixing angle is $\cos^2 \theta = v_g/c = 10^{-5}$ and the absorption length is of the order $L_{\text{abs}} \sim 100 \mu \text{m}$. The ratio $v_g/L_{\text{abs}}$ is the inverse time scale it takes a photon to travel one absorption length. The left-hand side of (16) demands that the rotation frequency is larger than this inverse decay time in order to see interesting physics.

There are numerous experimental systems which seem suitable for the implementation of the above suggested scheme. The choice of the systems is guided by the possibility to create strong and controllable interactions between individual polaritons to eventually explore effects such as the bosonic fractional quantum Hall effect. In addition to contact interactions of cold atoms in a trap it is possible to create long-range interactions by exploiting the interaction properties of the matter component of the polaritons.

**Bulk materials.**——A straightforward realization is to use rotating bulk media. Rare-earth-ion doped glasses exhibit long coherence times up to $T_2 = 82 \text{ ms}$ [26] and are proposed to be used for quantum computation [27]. Interactions can be created via electric dipole-dipole couplings involving different principal electronic states or via photonic nonlinearities. A table listing several experimental data of rare-earth-ion dopants can be found in [28]. Also $n$-doped semiconductors as GaAs [29] can create strong interactions by exploiting the Coulomb interaction between excitons, which constitute the matter component of the polaritons. However caused by the coupling of the electron spin to the nuclear spins coherence times are only of the order of several ns and thus too short. To overcome this one can use a $^{28}\text{Si}$-based host, which does not posses a nuclear spin, and shows coherence times up to 60 ms [30].

**Rotating optical lattices.**——In [31] the creation of a rotating optical lattice with frequencies up to several kHz is reported. With this one can uniformly rotate cold atoms or polar molecules as a bulk medium and at the same time take advantage of the high-precision techniques of this field. To create long-range interactions one may think of using Rydberg atoms or polar molecules. Recent works on Rydberg atoms report of the successful creation of Rydberg excitations in a BEC [32] with blockade radii of $r_b = 5.4 \mu \text{m}$. The lifetime of these systems is of the order of $\sim 100 \ \mu\text{s}$ [33]. Alternatively one could think of loading...
the optical lattice with polar molecules. There have been suggestions to create single-photon nonlinearities with these molecules [34] and investigations about the experimental feasibility [35] stating that lifetimes of about ~1 s are achievable. It should be noted that cold atoms and polar molecules can also be embedded in solid-state matrices [36], which can then be physically rotated.

To observe an artificial magnetic field for the DSP we suggest two possibilities. A first indication would be the observation of a Lorentz force acting on a slow-light polariton \( F = \frac{1}{m} (\hat{p}) \times B \). Shining in a probe beam with a small transverse extent along a propagation axis shifted from the axis of rotation will result in a small deflection of the incoming light pulse from its initial direction [37]. The deflection angle is given by \( \Delta \alpha = \omega_p \frac{\rho}{v_g} \), where \( \rho \) is the distance of the initial beam axis from the rotation axis. Using a stationary-light setup the outcome strongly depends on the initial mode profile. Creating the stationary DSPs using modes of the probe beams that are not eigenmodes of the angular momentum operator, e.g., higher Hermite-Gaussian modes, and releasing after a time will result in an image rotation. The angle of rotation is directly proportional to the storage time of the probe light inside the medium. If the initial state is a superfluid of DSPs [6] the artificial magnetic field leads to the formation of a vortex lattice. The structure of the lattice will be well visible upon releasing the stationary polaritons providing a convenient means to observe the lattice.

In summary we presented a possible scheme to create artificial gauge fields for photonic quasiparticles, the so-called dark-state polaritons. The size of the resulting effective magnetic field is large enough to create highly degenerate Landau levels. Observation of the artificial fields is possible by turning the stationary-light polaritons into slow-light polaritons [11] and detecting the transverse emission profile of light, which also allows a direct observation of phase profiles. We suggested several physical systems which, to our knowledge, seem suitable for the implementation of the above ideas. Polaritons not only have a number of technical advantages over electrons or cold atoms, they also offer to address new physical questions. E.g., making use of the fact that the polariton setup can be made an open system it should be possible to study a flux equilibrium, where dissipation drives the system automatically into highly correlated states [38]. Furthermore quantum Hall phenomena for multicomponent polaritons [13] including interspecies conversion are feasible.

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