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Evolution of Complex Systems and 1/f Noise: from Physics to Financial Markets

V. Gontis, B. Kaulakys, M. Alaburda and J. Ruseckas

Institute of Theoretical Physics and Astronomy, Vilnius University,
A. Goštauto 12, LT-2600 Vilnius, Lithuania; gontis@ktl.mii.lt

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Abstract. We introduce the stochastic multiplicative model of time intervals between the events, defining a multiplicative point process and analyze the statistical properties of the signal. Such a model system exhibits power-law spectral density $S(f) \sim 1/f^{\beta}$, scaled as power of frequency for various values of β between 0.5 and 2. We derive explicit expressions for the power spectrum and other statistics and analyze the model system numerically. The specific interest of our analysis is related with the theoretical modeling of the nonlinear complex systems exhibiting fractal behavior and self-organization.

Introduction

The power spectra of a large variety of different evolutional systems at low frequencies have 1/f behavior. 1/f noise has been observed in condensed matter systems, river discharge, DNA base sequence structure, cellular automata, traffic flow, economics, financial markets and other complex systems with the elements of self-organization (see, e.g., [1-3] and references herein). Considerable parts of such systems are fractal and their statistics exhibit scaling. The universality of 1/f noise suggests that it does not arise as a consequence of particular interaction but it is a characteristic signature of complexity. Computer experiments with various cellular automata and with other nonlinear systems illustrate how this might appear.

There are a number of the self-organized systems. One of the most popular of them is the sandpile model [4]. These nonlinear model systems were introduced also for explanation of 1/f noise as a result of the *self organized criticality*. The great interest of research in this direction is related with the complex behavior which mimics a noise with fractal characteristics. The noise, however, is originated from the nonlinear *deterministic* systems.

From our point of view it is possible to define a *stochastic* model system exhibiting fractal statistics and 1/f noise, as well. Such model system may represent the limiting behavior of the dynamical or deterministic complex systems, explaining the evolution of the complexity into chaotic regime. On the other hand, the stochastic point processes, analyzed in this paper, may be used for description of phenomena that occur as random sequences of events, exhibiting scaling of several statistics [5]. Considerable part of real stochastic sequences of events in physics, biomedicine, geophysics and economics are fractal. The proposed point process model of such systems has an evident physical meaning, because in the low frequency limit the real pulses may be represented as the point events.

The aim of this contribution is an introduction of the multiplicative stochastic model for the time interval between events in stochastic sequence, defining in such a way the multiplicative point process. We adopt the model of 1/f noise based on the Brownian motion of the time interval between subsequent pulses proposed in Refs. [3,6,7] introducing the stochastic multiplicative model of the interevent time [8]. The model exhibits the first order and the second order power-law statistics and serves as the theoretical description of the empirical trading activity in the financial markets [9].

Our specific interest is an analysis of the relation between the origin of the power-law distributions and the power-law correlations. Obviously, the multiplicative point process can be