Quantum anti-Zeno effect

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(Received 6 November 1996)

Prevention of a quantum system's time evolution by repetitive, frequent measurements of the system's state has been called the quantum Zeno effect (or paradox). Here we investigate theoretically and numerically the effect of repeated measurements on the quantum dynamics of multilevel systems that exhibit the quantum localization of classical chaos. The analysis is based on the wave function and Schrödinger equation, without introduction of the density matrix. We show how the quantum Zeno effect in simple few-level systems can be recovered and understood by formal modeling the effect of measurement on the dynamics by randomizing the phases of the measured states. This analysis is extended to investigate the dynamics of multilevel systems driven by an intense external force and affected by frequent measurements. We show that frequent measurements of such quantum systems results in delocalization of the quantum suppression of classical chaos. This result is the opposite of the quantum Zeno effect. The phenomenon of delocalization of the quantum suppression and restoration of quasi-classical time evolution of these systems, owing to repetitive frequent measurements, can therefore be called the *quantum anti-Zeno effect*. From this analysis we furthermore conclude that frequently or continuously observable quasiclassical systems evolve basically in a classical manner. [S1050-2947(97)07707-X]

PACS number(s): 03.65.Bz, 05.45.+b, 42.50.Lc

I. INTRODUCTION

Dynamics of a quantum system which is not being observed, can be described by the Schrödinger equation. In the von Neumann axiomatics of quantum mechanics it is postulated that any measurement gives rise to an abrupt change of the state of the system under consideration and projects it onto an eigenstate of the measured observable. The measurement process follows irreversible dynamics, e.g., due to coupling with the multitude of vacuum modes if spontaneous radiation is registered, and causes the disappearance of coherence of the system's state: the off-diagonal matrix elements of the density matrix decay, or the phases of the wave functions amplitudes are randomized.

It is known that a quantum system undergoes relatively slow (Gaussian, quadratic, or cosine type but not exponential) evolution soon after preparation or measurement [1]. Repetitive frequent observation of a quantum system can therefore inhibit the decay of an unstable [2] system and suppress dynamics of a system driven by an external field [3,4]. This inhibition, or even prevention, of the time evolution of the system by repeated frequent measurements from one eigenstate of an observable into a superposition of eigenstates is called the quantum Zeno effect (paradox) or the quantum watched pot [2–5]. Derivation and investigation of the quantum Zeno effect is usually based on the von Neumann's postulate of projection or reduction of the wave packet in the measurement process. However, the outcome of the variation of the quantum Zeno effect in a three-level system, originally proposed by Cook [3] and experimentally realized by Itano *et al.* [4], has been explained by Frerichs and Schenzle [6] on the basis of the standard three-level Bloch equations for the density matrix in the rotating-wave approximation with spontaneous relaxation. The quantum Zeno effect can thus be derived either from the *ad hoc* collapse hypothesis [2–4] or formulated in terms of irreversible quantum dynamics without additional assumptions, i.e., as the dynamical quantum Zeno effect [6,7]. Moreover, the postulate of "collapse of the wave function" models the actual measurement process only roughly [6].

Aharonov and Vardi [8] showed that frequent measurements can not only stop the quantum dynamics but can also induce time evolution of the observable system. These authors used the von Neumann projection postulate and predicted an evolution of the system along a presumed trajectory due to a sequence of measurements performed on states that slightly change from measurement to measurement. Altenmüller and Schenzle [9] have demonstrated that such a phenomenon can be explained by replacing the collapse hypothesis by that of an irreversible physical interaction.

It should be noted that most of the systems analyzed in the papers mentioned above consist of only a few (usually two or three) quantum states and are purely quantum systems. It is consequently of interest to investigate the influence of repeated frequent measurements on the evolution of multilevel quasiclassical systems, the classical counterparts of which exhibit chaos. It has been established [10–12] that chaotic dynamics of such systems, e.g., dynamics of nonlinear systems strongly driven by a periodic external field, is suppressed by the quantum interference effect and gives rise to quantum localization of the classical dynamics in the energy space of the system. The quantum localization phenomenon thus strongly limits the quantum motion. As stated above, repeated frequent measurements or continuous observation of a quantum system can inhibit its dynamics as well. It is therefore natural to expect that frequent measurements of suppressed system will result in additional freezing of the system's state.

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In connection with this question we refer to papers in which the influence of small external noise, environment, and measurement-induced effects on quantum chaos is analyzed (see [13–20] and references therein). The general conclusion of such investigations is that noise, interaction with the environment, and measurements induce decoherence, irreversibility, and delocalization. The direct link between measurements of the suppressed chaotic systems and the quantum Zeno effect has, however, to the best of our knowledge not yet been analyzed. We can only refer to papers [21] in which some preliminary relation between the quantum Zeno effect and the influence of repeated measurements on the dynamics of localized quantum systems is presented.

The purpose of the present study is to investigate theoretically and numerically the influence of frequent measurements on the evolution of multilevel quasiclassical systems.

Analysis of the measurement effect on the dynamics of a quantum system is usually performed with the aid of the density matrix formalism. The investigation of the quantum dynamics of a multilevel system affected by repeated measurements is, however, very difficult analytically and time consuming in numerical calculations. Analysis based on the wave function and Schrödinger equation is considerably more tractable and transparent. We first show how the quantum Zeno effect in a few-level system can be described in terms of the wave function and Schrödinger equation without introducing the density matrix and how the measurements can be incorporated in the equations of motion.

We then apply the same method to the analysis of the dynamics of a multilevel system affected by repeated frequent measurements. We show that repetitive measurements on a multilevel system with quantum suppression of classical chaos results in delocalization of the superposition of state, and restoration of chaotic dynamics. Since this effect is the reverse of the quantum Zeno effect we call this phenomenon the *quantum anti-Zeno effect*.

II. DYNAMICS OF A TWO-LEVEL SYSTEM

We consider the simplest quantum-dynamical process and the influence of frequent measurements on the outcome of the dynamics. Time evolution of the amplitudes $a_1(t)$ and $a_2(t)$ of the two-state wave function

$$\Psi = a_1(t)\Psi_1 + a_2(t)\Psi_2 \tag{2.1}$$

of the system in the resonance field (in the rotating-wave approximation) or of the spin-one-half system in a constant magnetic field can be represented as

$$\begin{aligned} a_1(t) &= a_1(0)\cos\tfrac{1}{2}\Omega t + i a_2(0)\sin\tfrac{1}{2}\Omega t,\\ a_2(t) &= i a_1(0)\sin\tfrac{1}{2}\Omega t + a_2(0)\cos\tfrac{1}{2}\Omega t, \end{aligned} \tag{2.2}$$

where Ω is the Rabi frequency. We introduce a matrix $\mathbf{A}$ representing time evolution during the time interval τ, between time moments $t=k\tau$ and $t=(k+1)\tau$ with k integer, and rewrite Eq. (2.2) in the mapping form

$$\begin{pmatrix} a_1(k+1) \\ a_2(k+1) \end{pmatrix} = \mathbf{A} \begin{pmatrix} a_1(k) \\ a_2(k) \end{pmatrix}, \tag{2.3}$$

where the evolution matrix $\mathbf{A}$ is

$$\mathbf{A} = \begin{pmatrix} \cos\varphi & i\sin\varphi \\ i\sin\varphi & \cos\varphi \end{pmatrix}, \quad \varphi = \frac{1}{2}\Omega\tau. \tag{2.4}$$

The evolution of the amplitudes from time $t=0$ to $t=T=n\tau$ can be expressed as

$$\begin{pmatrix} a_1(n) \\ a_2(n) \end{pmatrix} = \mathbf{A}^n \begin{pmatrix} a_1(0) \\ a_2(0) \end{pmatrix}. \tag{2.5}$$

One can calculate matrix $\mathbf{A}^n$ by diagonalizing the matrix $\mathbf{A}$:

$$\mathbf{A}^n = \begin{pmatrix} \cos n\varphi & i\sin n\varphi \\ i\sin n\varphi & \cos n\varphi \end{pmatrix}, \tag{2.6}$$

where $n\varphi = \frac{1}{2}\Omega T$.

Equations (2.2)–(2.6) represent the time evolution of the system without intermediate measurements in the interval 0–T. If at $t=0$ the system was in the state Ψ_1, i.e., $a_1(0)=1$ and $a_2(0)=0$, and if $\Omega T=\pi$, then at the moment $t=T$ we will certainly find the system in the state Ψ_2, i.e., we have $|a_1(T)|^2=0$ and $|a_2(T)|^2=1$, a certain (with probability 1) transition between the states. Such quantum dynamics without intermediate measurements is regular and coherent for all time until the final measurement.

Consider now the dynamics of a system subjected to intermediate measurements at intervals τ. Measurement of the system's state, at time $t=k\tau$ projects the system onto the state Ψ_1 with probability $p_1(k)=|a_1(k)|^2$ or onto the state Ψ_2 with probability $p_2(k)=|a_2(k)|^2$. After the measurement we know the probabilities $p_1(k)$ and $p_2(k)$ but we have no information about the phases $\alpha_1(k)$ and $\alpha_2(k)$ of the amplitudes

$$a_1(k) = |a_1(k)|e^{i\alpha_1(k)}, \quad a_2(k) = |a_2(k)|e^{i\alpha_2(k)}, \tag{2.7}$$

i.e., the phases $\alpha_1(k)$ and $\alpha_2(k)$ after measurement are random. Randomization of the phases after a measurement can also be confirmed by analysis of the definite measurement process, e.g., in the **V**-shape three-level configuration with the spontaneous transition to the ground state [3–7]. Every measurement of the system's state results in mutually uncorrelated phases $\alpha_1(k)$ and $\alpha_2(k)$. After the full measurement of the system's state these phases are also uncorrelated with the phases before the measurement. For this reason, according to the measurement postulate, the outcome of the measurement does not depend on the phases of the amplitudes in the expansion of the wave function in terms of the eigenfunctions of the measured observable. The interference terms in the expressions derived below hence do not affect the transition probabilities between the eigenstates.

Now we derive equations for the transition probabilities between the states in the case of evolution with intermediate measurements. From Eqs. (2.3) and (2.4) we have

$$\begin{aligned}|a_1(k+1)|^2 &= |a_1(k)|^2\cos^2\varphi + |a_2(k)|^2\sin^2\varphi \\ &\quad + |a_1(k)a_2(k)|\sin[\alpha_1(k)-\alpha_2(k)]\sin 2\varphi, \\ |a_2(k+1)|^2 &= |a_1(k)|^2\sin^2\varphi + |a_2(k)|^2\cos^2\varphi \\ &\quad - |a_1(k)a_2(k)|\sin[\alpha_1(k)-\alpha_2(k)]\sin 2\varphi.\end{aligned} \tag{2.8}$$

After a measurement at time $t=k\tau$ the phase difference $\alpha_1(k)-\alpha_2(k)$, according to the above statement, is random and the contribution of the last term in expressions (2.8) to the average of the large number of iterations equals zero. The equations for the probabilities hence are

$$\begin{pmatrix} p_1(k+1) \\ p_2(k+1)\end{pmatrix} = \mathbf{M}\begin{pmatrix} p_1(k) \\ p_2(k)\end{pmatrix}, \tag{2.9}$$

where

$$\mathbf{M} = \begin{pmatrix} \cos^2\varphi & \sin^2\varphi \\ \sin^2\varphi & \cos^2\varphi \end{pmatrix} \tag{2.10}$$

is the evolution matrix for the probabilities. The full evolution from the initial time $t=0$ until $t=T$ with $(n-1)$ equidistant intermediate measurements is described by the equation

$$\begin{pmatrix} p_1(n) \\ p_2(n)\end{pmatrix} = \mathbf{M}^n\begin{pmatrix} p_1(0) \\ p_2(0)\end{pmatrix}. \tag{2.11}$$

The result of the calculation of the matrix $\mathbf{M}^n$ by the method of diagonalization of the matrix $\mathbf{M}$ is

$$\mathbf{M}^n = \frac{1}{2}\begin{pmatrix} 1+\cos^n 2\varphi & 1-\cos^n 2\varphi \\ 1-\cos^n 2\varphi & 1+\cos^n 2\varphi \end{pmatrix}. \tag{2.12}$$

From Eqs. (2.11) and (2.12) we recover the quantum Zeno effect obtained by the density matrix technique [3–6]: if initially the system is in the state Ψ_1, then the result of the evolution until the time $T=n\tau=\pi/\Omega$ (after the π pulse) with $(n-1)$ intermediate measurements is characterized by the probabilities $p_1(T)$ and $p_2(T)$ for finding the system in the states Ψ_1 and Ψ_2, respectively:

$$\begin{aligned} p_1(T) &= \tfrac{1}{2}(1+\cos^n 2\varphi) \simeq \tfrac{1}{2}(1+e^{-\pi^2/2n}) \simeq 1-\frac{\pi^2}{4n} \to 1, \\ p_2(T) &= \tfrac{1}{2}(1-\cos^n 2\varphi) \simeq \tfrac{1}{2}(1-e^{-\pi^2/2n}) \simeq \frac{\pi^2}{4n} \to 0, \quad n\to\infty. \end{aligned} \tag{2.13}$$

We see that Eqs. (2.11)–(2.13) represent inhibition of the quantum dynamics by measurements and coincide with those obtained by the density-matrix technique [3–6]. This also confirms correctness of the proposition that the act of measurement can be represented as randomization of the amplitudes' phases. Further we will use this proposition and the same method to analyze the influence of repeated measurements on the quantum dynamics of multilevel systems, the classical counterparts of which exhibit chaos. We restrict ourselves to systems of one degree of freedom strongly driven by a periodic force. The investigation is also based on the mapping equations of motion for such systems.

III. QUANTUM MAPS FOR MULTILEVEL SYSTEMS

In general, the classical equations of motion are nonintegrable and the Schrödinger equation for strongly driven systems cannot be solved analytically. The mapping forms of the classical and quantum equations of motion however greatly facilitates the investigation of stochasticity and quantum-classical correspondence for the chaotic dynamics. From the standpoint of understanding the effect of the measurements on the dynamics of multilevel systems, the region of large quantum numbers is of greatest interest. Here we can use the quasiclassical approximation when convenient variables are the angle θ and action I. Transition from the classical to a quantum (quasiclassical) description can be undertaken by replacing I with the operator $\hat{I}=-i(\partial/\partial\theta)$ [22,23]. (We use the system of units with $\hbar=1$.) One of the simplest cases in which dynamical chaos and its quantum localization can be observed is a system with one degree of freedom described by the unperturbed Hamiltonian $H_0(I)$ and driven by periodic kicks. The full Hamiltonian H of the driven system can be represented as

$$H(I,\theta,t) = H_0(I) + k\cos\theta \sum_j \delta(t-j\tau), \tag{3.1}$$

where τ and k are the period and strength of the perturbation, respectively.

The intrinsic frequency of the unperturbed system is $\Omega=dH_0/dI$. In particular, for a linear oscillator we have $H_0=\Omega I$. The Hamiltonian $H_0=I^2/2$ describes the widely investigated rotator, which results in the so-called standard map [12,14], while the Hamiltonian (3.1) with $H_0=\omega/[2\omega(I_0+I)]^{1/2}$ and $k=2\pi bF/\omega^{5/3}$ ($b\simeq 0.411$) models a highly excited atom in a monochromatic field of strength F and frequency ω [23,25–27].

Integration of the classical equations of motion for the Hamiltonian (3.1) over the perturbation period τ leads to the classical map for the action and angle

$$\begin{aligned} I_{j+1} &= I_j + k\sin\theta, \\ \theta_{j+1} &= \theta_j + \tau\Omega(I_{j+1}). \end{aligned} \tag{3.2}$$

For the rotator, the unperturbed frequency is $\Omega(I_{j+1})=I_{j+1}$ and the map (3.2) coincides with the standard map investigated in great detail [12,22,24].

For the derivation of the quantum equations of motion we expand the state function $\psi(\theta,t)$ of the system in quasiclassical eigenfunctions, $\varphi_n(\theta)=e^{in\theta}/\sqrt{(2\pi)}$, of the Hamiltonian H_0,

$$\psi(\theta,t) = (2\pi)^{-1/2}\sum_n a_n(t) i^{-n} e^{-in\theta}. \tag{3.3}$$

Here, the phase factor i^{-n} is introduced for maximal simplification of the quantum map. Integration of the Schrödinger equation over the period τ, leads to the following maps for the amplitudes before the kicks [21,23]:

$$a_m(t_{j+1}) = e^{-iH_0(m)\tau} \sum_n a_n(t_j) J_{m-n}(k), \quad t_j = j\tau, \tag{3.4}$$

where the $J_m(k)$ are Bessel functions.

The form (3.4) of the map for the quantum dynamics is rather common: similar maps can be derived for monochromatic perturbations as well, e.g., for an atom in a microwave field [23,27]. A particular case of map (3.4), corresponding to the model of a quantum rotator $H = I^2/2$, has been investigated comprehensively with the aim of determining the relationship between classical and quantum chaos [12,22,24]. It has been shown that at the onset of dynamical chaos at $K \equiv \tau k > K_c = 0.9816$, motion with respect to I is not bounded and is a kind of diffusion in the classical case, while in the quantum description diffusion with respect to m is bounded, i.e., the diffusion ceases after some time and the state of the system is localized exponentially. The exponential localization length λ of the quantum state is usually defined as

$$\lim_{N\to\infty} |a_m(N\tau)|^2 \sim \exp\left(-\frac{2|m-m_0|}{\lambda}\right), \tag{3.5}$$

where m_0 is the initial action. It has been shown [10–12] that for a quantum rotator the localization length is $\lambda \simeq k^2/2$. The effect of quantum limitation of dynamical chaos is extremely interesting and important. It reveals itself in many quantum systems, for which the classical counterparts exhibit chaos. It should be noted that for the rotator the exact quantum description coincides with the quasiclassical one.

The classical dynamics of the system described by map (3.2) in the case of global distinct stochasticity is diffusion-like with the diffusion coefficient in I space

$$B(I) = \overline{(\Delta I)^2}/2\tau = k^2/4\tau. \tag{3.6}$$

From Eqs. (3.4) we obtain the transition probabilities $P_{n,m}$ between states n and m during the period τ

$$P_{n,m} = J^2_{m-n}(k). \tag{3.7}$$

Using the expression $\Sigma_n n^2 J_n^2(k) = k^2/2$ and approximation of the uncorrelated transitions we can formally evaluate the local quantum diffusion coefficient in n space [21,25,26],

$$B(n) = \frac{1}{2\tau} \sum_m (m-n)^2 J^2_{m-n}(k) = \frac{k^2}{4\tau}. \tag{3.8}$$

The expression for the local quantum diffusion coefficient hence coincides with the classical equation (3.6).

It turns out, however, that such quantum diffusion takes place only for some finite time $t \le t^* \simeq \tau k^2/2$ [28], after which an essential decrease of the diffusion rate is observed. Such behavior of quantum systems in the region of strong classical chaos is called "quantum suppression of classical chaos" [10,11]. This phenomenon turns out to be typical for models (3.1) with nonlinear Hamiltonians $H_0(I)$ as well as for other quantum systems. The diffusion coefficient (3.8) derived in the approximation of uncorrelated transitions (3.7) thus does not describe the true quantum dynamics in energy space.

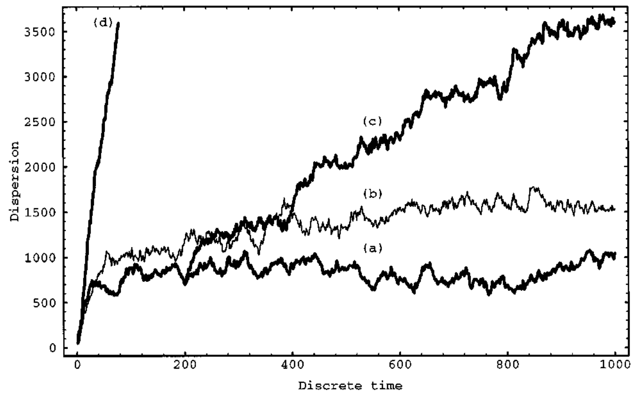

FIG. 1. Dependence of the dimensionless momentum dispersion, $\langle (m-m_0)^2 \rangle$, as defined by Eq. (4.1) for the quantum rotator with $m_0 = 500$, $\tau = 1$, and $k = 10$ on the discrete dimensionless time j for the dynamics according to Eq. (3.4): (a) without the intermediate measurements, (b) with measurements of the initial state, φ_{500}, after every kick, (c) with measurements of all states every 200 kicks, and (d) with measurements of all states after every kick.

The quantum interference effect is essential for such dynamics and results in the quantum evolution being quantitatively different from the classical motion. Quantum equations of motion, i.e., the Schrödinger equation and the maps for the amplitudes, are linear with respect to the wave function and probability amplitudes. The quantum interference effect therefore manifests itself even for the quantum dynamics of the systems, the classical counterparts of which are described by nonlinear equations; chaotic dynamics of the latter exhibit dynamical chaos. On the other hand, quantum equations of motion are very complex as well. The Schrödinger equation is a partial differential equation with coordinate- and time-dependent coefficients, while the system of equations for the amplitudes is infinite. Moreover, for the nonlinear Hamiltonian $H_0(m)$, the phase increments $H_0(m)\tau$ during the free motion between two kicks, while reduced to the basic interval $[0,2\pi]$, are pseudorandom quantities as functions of the state's quantum number m. This causes a very complicated and irregular quantum dynamics of the classically chaotic systems. We observe not only very large and apparently irregular fluctuations of probabilities of the states' occupation but also almost irregular fluctuations in time of the momentum dispersion [see curves (a) in Figs. 1 and 2].

The quantum dynamics of such systems driven by an external periodic force is, however, coherent and the evolution of the amplitudes $a_m(t_{j+1})$ in time is linear: they are defined by the linear map (3.4) with time-independent coefficients. The nonlinearity of the Hamiltonian $H_0(I)$, being the reason for the classical chaos, causes the pseudorandom nature of the increments of the phases $H_0(m)\tau$ as functions of the state's number m (but constant in time). These increments of the phases remain the same for each kick. The dynamics of the amplitudes $a_m(t_{j+1}) = |a_m(t_{j+1})| e^{i\alpha_m(t_{j+1})}$ and of their

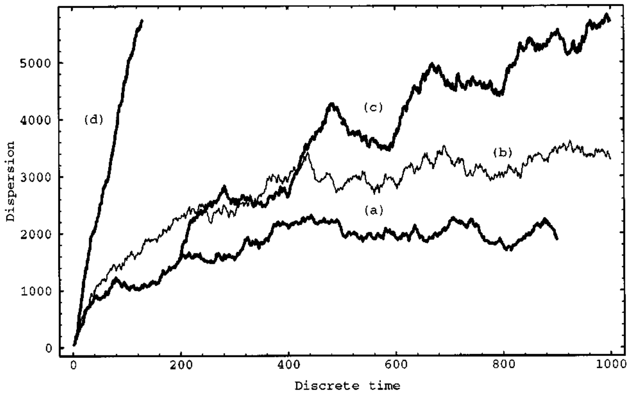

FIG. 2. Same as in Fig. 1 but for the system with random distribution of energy levels, i.e., for random phases $H_0(m)\tau$ in Eqs. (3.4) defined as $2\pi g_m$, where g_m is a sequence of random numbers that are uniformly distributed in the interval [0,1].

phases $\alpha_m(t_{j+1})$ is thus strongly deterministic and nonchaotic, but very complicated and apparently irregular. For instance, the phases $\alpha_m(t_{j+1})$ are phases of the complex amplitudes $a_m(t_{j+1})$, which are linear combinations (3.4) of the complex amplitudes $a_n(t_j)$ before the preceding kick, with the pseudorandom coefficients $e^{-iH_0(m)\tau}J_{m-n}(k)$. Nevertheless, the iterative equation (3.4) is a *linear transformation with coefficients regular in time.* That is why we observe for such dynamics the quasiperiodic reversibility in the time evolution [12] with the quantum localization of the pseudochaotic motion.

In Ref. [23] it has been demonstrated that this peculiarity of the pseudochaotic quantum dynamics is indeed due to the pseudorandom nature of the phases $H_0(m)\tau$ in Eq. (3.4) as a function of the eigenstate's quantum number m (but not of the kick's number j). Replacing the multipliers $\exp[-iH_0(m)\tau]$ in Eq. (3.4) by the expressions $\exp[-i2\pi g_m]$, where g_m is a sequence of random numbers that are uniformly distributed in the interval [0,1], we observe the quantum localization as well [23]. The essential point here is the independence of the phases $H_0(m)\tau$ or $2\pi g_m$ on the step of iteration j or time t. This is the basic difference from the randomness of the phases due to measurements under consideration in Sec. IV.

IV. INFLUENCE OF REPETITIVE MEASUREMENTS ON THE QUANTUM DYNAMICS

Each measurement of the system's state projects the state onto one of the energy states φ_m with definite m. Therefore, if we make a measurement of the system after the kick j but before the next kick $(j+1)$, we will find the system in the state φ_m with probability $p_m(j)=|a_m(t_j)|^2$.

In principle, such a measurement can be performed as in the experiment of Itano *et al.* [4], i.e., by short-impulse laser excitation of the system from state φ_m to some higher state followed by the irreversible return of the system to the same state φ_m, with registration of the state's population by photon counting. After the measurement of the population of the state φ_m the probability of finding the system in the state φ_m coincides with that before the measurement. There is, however, no interference between the amplitude $\widetilde{a}_m(t_j)$ of the state φ_m after the measurement and the amplitudes of other states, $a_n(t_j)$, i.e., the cross terms containing the amplitude $\widetilde{a}_m(t_j)$ in the expressions for probabilities vanish. Interference between the unmeasured states remains, however, and the cross terms containing the amplitudes of the unmeasured states do not vanish.

In the calculation of the system's dynamics the influence of measurements can be taken into account in the same way as in Sec. II, i.e., by randomizing the phases of the amplitudes after the measurement of the appropriate state's population. The phases of amplitudes after the measurements are completely random and uncorrelated with the phases before the measurements, after another measurement, or with the phases of other measured or unmeasured states. After a full measurement of the system after a kick j, all phases of the amplitudes $a_m(t_j)$ therefore are random. This full measurement of the system's state therefore influences the further dynamics of the system through randomization of the phases of the amplitudes (see Sec. II for analogy). This fact can be expressed by replacement in Eqs. (3.4) of the amplitudes $a_m(t_{j+1})$ by the amplitudes $e^{i\beta_m(t_{j+1})}a_m(t_{j+1})$ with random phases $\beta_m(t_{j+1})$. The essential point here is that the phases $\beta_m(t_{j+1})$ are different, uncorrelated for the different measurements, i.e., for different time moments of the measurement t_{j+1}. This is the principal difference between the random phases $\beta_m(t_{j+1})$ and the phases $H_0(m)\tau$ in Eqs. (3.4), which are pseudorandom variables as functions of the eigenstate's quantum number m (but not of the time t_{j+1}).

Introducing the appropriate random phases we can thus analyze the influence on the system's dynamics of full measurements of the system's state performed after every kick, after every N kicks, or of measurements of the population probabilities just of some states, e.g., only the initial state. There is no need to measure more frequently than after every kick because the results of subsequent measurements before the next kick repeat the results of measurements after the last kick.

Instead of representing the detailed quantum dynamics expressed as the evolution of all amplitudes in the expansion (3.3) of the wave function we can represent only dynamics of the momentum dispersion

$$\langle (m_j-m_0)^2\rangle = \sum_m (m-m_0)^2|a_m(t_j)|^2, \tag{4.1}$$

where m_0 is the initial momentum quantum number. Such a representation of the dynamics is simpler, more easily pictured, and more readily compared with classical dynamics.

In Figs. 1 and 2 we show the results of numerical analysis of the influence of measurements of the system's state on the quantum dynamics of a rotator and of a system with random distribution of energy levels, i.e., for random phases $H_0(m)\tau$ in Eqs. (3.4) as a function of the eigenstate's quantum number m. We see that quantum diffusionlike dynamics of the systems without measurements, represented by curves (a), after sufficiently short time $t^* \simeq \tau k^2/2$ (of the order of 50τ in our case) ceases. For time $t \gg t^*$ the monotonic increase of the momentum dispersion $\langle (m-m_0)^2 \rangle \simeq 2Bt = (k^2/2\tau)t$ for time $t \ll t^*$ turns into the stationary distribution (on average for the time interval $\Delta t \gg t^*$) with the momentum dispersion $\langle (m_{st}-m_0)^2 \rangle \simeq \lambda^2/2 \simeq k^4/8$. This is a demonstration of the effect of quantum suppression of classical chaos.

In the case of measurement of the population of only the initial state φ_{500} after every kick [which technically is achieved by introduction of the random phase $\beta_{500}(t_{j+1})$, after every kick j], we observe monotonic, though slow, increase of the momentum dispersion for very long time, until $t \sim 600\tau$ in our case [curves (b) in Figs. 1 and 2]. After such time the population of the initial state on the average becomes very small and measurement of this state's population almost does not influence the system's dynamics.

The dynamics with measurements of all states every 200 kicks represented by curves (c) is staircaselike: fast increase of the momentum dispersion after the immediate measurement turns into quantum suppression of the diffusionlike motion for $\Delta t \gg t^*$ until the next measurement destroys the quantum interference and induces the succeeding diffusionlike motion.

The quantum dynamics of the kicked rotator or some other system with measurements of all states' populations after every kick, represented by curves (d), is essentially classical-like: the momentum dispersion increases linearly in time with the classical diffusion coefficient (3.6) for all time of the calculation.

Theoretically such differences of dynamics can be understood from the iterative equations for the momentum dispersion

$$\langle (m_{j+1}-m_0)^2 \rangle = \sum_m (m-m_0)^2 |a_m(t_{j+1})|^2, \tag{4.2}$$

where

$$|a_m(t_{j+1})|^2 = \sum_{n,n'} J_{m-n}(k) J_{m-n'}(k) a_n(t_j) a^*_{n'}(t_j). \tag{4.3}$$

Substitution of Eq. (4.3) into Eq. (4.2) yields

$$\begin{aligned}\langle (m_{j+1}-m_0)^2 \rangle = &\sum_{m,n} (m-m_0)^2 J^2_{m-n}(k) |a_n(t_j)|^2 \\ &+ 2\sum_{m,n} \sum_{n'<n} (m-m_0)^2 J_{m-n}(k) J_{m-n'}(k) \\ &\times Re[a_n(t_j) a^*_{n'}(t_j)]. \end{aligned} \tag{4.4}$$

For the random phase differences of the amplitudes $a_n(t_j)$ and $a^*_{n'}(t_j)$ with $n' \neq n$ (after the measurement of the system's state), the second term in Eq. (4.4) on average equals zero (see Sec. II for clarification). From Eq. (4.4) we then have

$$\begin{aligned}\langle (m_{j+1}-m_0)^2 \rangle &= \sum_n |a_n(t_j)|^2 \sum_m (m-m_0)^2 J^2_{m-n}(k) \\ &= \sum_m |a_m(t_j)|^2 \left(m^2 - m_0^2 + \frac{k^2}{2} \right) \\ &= \langle (m_j - m_0)^2 \rangle + \frac{k^2}{2}. \end{aligned} \tag{4.5}$$

In the derivation of Eq. (4.5) we have used the summations

$$\sum_m m J^2_{m-n}(k) = 0 \quad \text{and} \quad \sum_m m^2 J^2_{m-n}(k) = n^2 + \frac{k^2}{2}.$$

According to Eq. (4.5) for the uncorrelated phases of the amplitudes $a_n(t_j)$ and $a^*_{n'}(t_j)$ with $n' \neq n$, the dispersion of the momentum as a result of every kick therefore increases on average by magnitude $k^2/2$, just as for classical dynamics. For the dynamics of isolated quantum systems in the absence of measurements or unpredictable interaction with the environment, the second term of Eq. (4.4) compensates (on average for sufficiently large time intervals $\Delta t \gg t^*$) the first term of Eq. (4.4), due to the quantum interference between the amplitudes of different states arisen from the same initial states' superposition. Thus the quantum suppression of dynamics may be observed.

Similar analysis can be used as well for the investigation of the influence of measurements on the quantum dynamics of another quantum systems with quantum localization of the classical chaos.

As has already been stated above, the influence of repetitive measurements on quantum dynamics is closely related to the effect of unpredictable interactions between the system and the environment. It should be noticed that in general, for the analysis of the measurement effect and to facilitate the comparison between quantum and classical dynamics of chaotic systems, it is convenient to employ the Wigner representation $\rho_W(x,p,t)$ of the density matrix [19,29]. The Wigner function of a system with the Hamiltonian of form $H = p^2/2m + V(x,t)$ evolves according to the equation

$$\begin{aligned}\frac{\partial \rho_W}{\partial t} &= \{H, \rho_W\}_M \equiv \{H, \rho_W\} \\ &+ \sum_{n \geq 1} \frac{\hbar^{2n}(-1)^n}{2^{2n}(2n+1)!} \frac{\partial^{2n+1} V}{\partial x^{2n+1}} \frac{\partial^{2n+1} \rho_W}{\partial p^{2n+1}}, \end{aligned} \tag{4.6}$$

where $\{\cdots\}_M$ and $\{\cdots\}$ denote the Moyal and Poisson brackets, respectively. The terms in Eq. (4.6) containing Planck's constant and higher derivatives represent the quantum corrections to the classical dynamics generated by the Poisson brackets. In the region of regular dynamics, one can neglect the quantum corrections for very long times if the characteristic actions of the system are large. For classically chaotic motion, the exponential instabilities lead to the development of the fine structure of the Wigner function and exponential growth of its derivatives. As a result, the quantum corrections become significant after a relatively short time even for macroscopic bodies [19,28]. The extremely small additional diffusionlike terms in Eq. (4.6), which reproduce the effect of interaction with the environment or frequent measurements, prohibit development of the Wigner function's fine structure and remove barriers posed by classical chaos for the correspondence principle [19,29].

V. CONCLUSIONS

From the above analysis we can conclude that the influence of repetitive measurements on the dynamics of a quasiclassical multilevel systems with quantum suppression of classical chaos is opposite to that of the few-level quantum system. Repetitive measurements of multilevel systems result in delocalization of the states' superposition and acceleration of the chaotic dynamics. In the limit of frequent full measurements of the system's state the quantum dynamics approaches classical motion, which is the opposite of the quantum Zeno effect: inhibition or even prevention of time evolution of the system from an eigenstate of an observable into a superposition of eigenstates by repeated frequent measurements. We can therefore call this phenomenon the ''quantum anti-Zeno effect.''

It should be noted that the same effect can be derived without the *ad hoc* collapse hypothesis but from the quantum theory of irreversible processes, in analogy with the method used in [6,9]. Even the simplest detector follows irreversible dynamics due to coupling to the multitude of vacuum modes, which results in the randomization of the quantum amplitudes' phases, decay of the off-diagonal matrix elements of the density matrix, and smoothing of the fine structure of the Wigner distribution function, i.e., just what is needed to obtain the classical equations of motion.

The quantum-classical correspondence problem caused by chaotic dynamics thus is closely related to the old problem of measurement in quantum mechanics. In the case of frequent measurements or unpredictable interaction with the environment, the quantum dynamics of quasiclassical systems approaches classical-like motion.

The research described in this publication was made possible in part by support of the Alexander von Humboldt Foundation.

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