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PARAMETRIC GENERATION OF LIGHT X WAVES BY BESSEL BEAM PUMP *

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It is shown that Bessel beam pump in optical parametric generator supports an existence of two different X wave modes at different propagation velocities. The phase matching possibility of Bessel beam pump with a X wave propagating in opposite direction is revealed.

Keywords: Bessel beam, X wave, optical parametric generation

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Spatial and temporal localization of light energy is of great importance for various applications. The diffraction-free and dispersion-free propagation of pulsed beams (usually called as X waves) can be achieved both in linear [1–3] and nonlinear media. In a quadratic medium X waves are formed spontaneously through a trigger mechanism of conical emission via mismatched second harmonic generation [4–9]. An appearance of conical emission is a result of noncollinear interactions which are strictly controlled by phase matching conditions of different components of spatial-temporal spectrum of the waves. It was demonstrated that angular dispersion of the waves excited in optical parametric generator (OPG) by quasimonochromatic plane pump wave corresponds to angular dispersion of X waves [10, 11]. In this paper we reveal an existence of different X wave modes in OPG pumped by quasimonochromatic Bessel beam.

An X wave can be represented as a superposition of Bessel beams, the frequencies ω, wave vectors k, and half-cone angles ψ of which are related in dispersive medium by an equation

$$k\left(\omega\right)\cos\psi\left(\omega\right)=\frac{\omega}{V}+\gamma\,, \qquad (1)$$

where V is the velocity of X wave and γ is an arbitrary constant [1]. The phase matching conditions of X waves interacting with a quasimonochromatic Bessel beam pump in the nonlinear crystal can be obtained

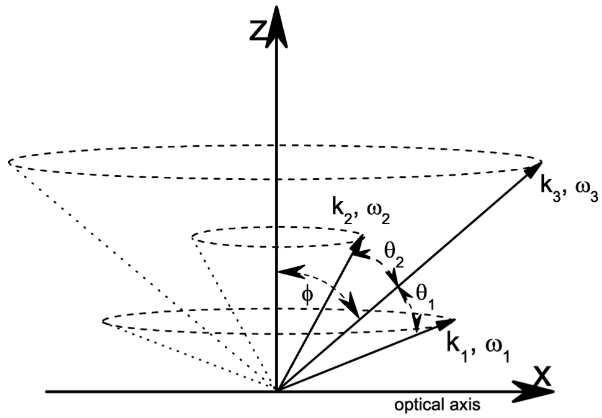

Fig. 1. Schematic depiction of noncollinear phase matching of three plane monochromatic waves in nonlinear crystal.

from the conditions of noncollinear phase matching of plane monochromatic waves, if the requirements of proper angular dispersion of X waves are taken into account. We assume that Bessel pump beam (frequency ω_3, half-cone angle ϕ) is propagating along axis z that is perpendicular to an optical axis (x) of the uniaxial negative crystal, Fig. 1. As a result, in the paraxial approximation within accuracy $\sim\ \phi^3$ the interacting waves for type I phase matching can be considered as axially symmetric. Then, the phase matching conditions can be written as

$$\omega_1+\omega_2=\omega_3\,, \qquad (2)$$

$$k_1\sin\left(\phi+\theta_1\right)+k_2\sin\left(\phi+\theta_2\right)=k_3\sin\phi\,, \qquad (3)$$

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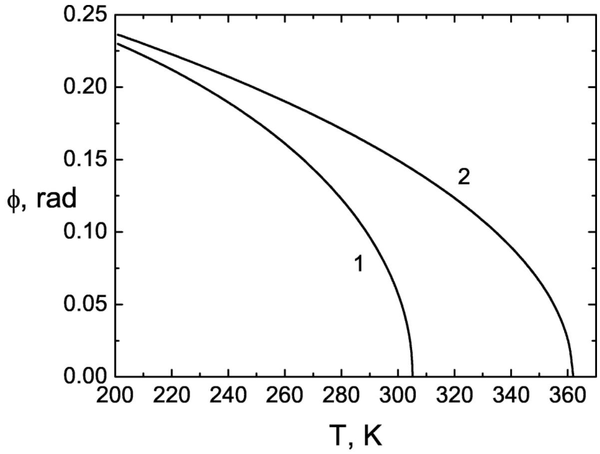

Fig. 2. Dependence of the direction of collinear degenerate parametric interaction (angle ϕ) on crystal temperature for $LiNbO_3$ (*1*, $\lambda_3 = 0.532$ μm) and KDP (*2*, $\lambda_3 = 0.266$ μm) crystals.

$$k_1 \cos(\phi + \theta_1) + k_2 \cos(\phi + \theta_2) = k_3 \cos\phi + \Delta k\,, \tag{4}$$

where the subscripts number the waves, and Δk is the phase-mismatch for arbitrary chosen frequencies ω_1, ω_2.

Further we suppose that angle ϕ corresponds to the direction of collinear degenerate parametric interaction. In this case $\omega_1 = \omega_2 = \omega_0 = \omega_3/2$ and $k_3 = 2k_0$, where $k_0 = k(\omega_0)$. The dependences of the angle ϕ on temperature in $LiNbO_3$ and KDP crystals are shown in Fig. 2 at pump wavelengths 0.532 and 0.266 μm, respectively. The results were obtained by use of refractive-index data presented in [12]. We note, that angle $\phi = 0$ corresponds to noncritical phase matching of interacting waves, and in this case the pump beam is a plane wave.

We suppose that frequencies ω_1 and ω_2 can be written as $\omega_1 = \omega_0 + \Delta\omega$ and $\omega_2 = \omega_0 - \Delta\omega$, where $\Delta\omega$ is a frequency shift with respect to ω_0. Then, the dispersive relations of two X waves take a form (see Eq. (1))

$$k_1 \cos(\phi + \theta_1) = \frac{\Delta\omega}{V^{(1)}} + k_0 \cos\phi\,, \tag{5}$$

$$k_2 \cos(\phi + \theta_2) = -\frac{\Delta\omega}{V^{(2)}} + k_0 \cos\phi\,. \tag{6}$$

As a result, for phase mismatch Δk of X waves we find $\Delta k = \Delta\omega\left(1/V^{(1)} - 1/V^{(2)}\right)$. Obviously, the most effective excitation of X waves from quantum noise level in OPG should occur at $\Delta k = 0$ ($V^{(1)} = V^{(2)} = V$). At $\Delta k = 0$ the angle ϕ in Eqs. (3) and (4) can be excluded, and the phase matching conditions of noncollinear interaction can be rewritten as

$$k_1 \sin\theta_1 + k_2 \sin\theta_2 = 0\,, \tag{7}$$

$$k_1 \cos\theta_1 + k_2 \cos\theta_2 = k_3\,. \tag{8}$$

Then, in the paraxial approximation (small angles θ_1, θ_2) the angular dispersion of excited waves is given by

$$\theta_1^2 \approx 2\frac{k_2(k_1 + k_2 - k_3)}{k_1(k_1 + k_2)}\,, \quad \theta_2 \approx -\frac{k_1}{k_2}\theta_1\,. \tag{9}$$

Further, an expansion of the wave vectors k_1 and k_2 into a Taylor series $k(\omega_0 \pm \Delta\omega) \approx k_0 \pm \Delta\omega/u_0 + g_0/2\,(\Delta\omega)^2$ gives

$$\theta_1 \approx \pm\sqrt{\frac{g_0}{k_0}}\Delta\omega\,, \quad \theta_2 \approx -\theta_1\,, \tag{10}$$

where u_0 and g_0 are group velocity and group velocity dispersion coefficient, respectively. So, in the case of degenerate collinear interaction of central frequencies of the waves, the type I phase matching causes a linear angular dispersion of excited pulsed beams. The obtained angular dispersion, Eq. (10), should comply with an angular dispersion of X waves. In paraxial approximation, after expansion of the wave vectors k_1 and k_2 into Taylor series, Eqs. (5) and (6) can be written as

$$g_0(\Delta\omega)^2 + 2\Delta\omega\left(\frac{\cos\phi}{u_0} - \frac{1}{V}\right) - 2\frac{\Delta\omega}{u_0}\theta_1\sin\phi - k_0\theta_1^2\cos\phi - 2k_0\theta_1\sin\phi = 0\,, \tag{11}$$

$$g_0(\Delta\omega)^2 - 2\Delta\omega\left(\frac{\cos\phi}{u_0} - \frac{1}{V}\right) + 2\frac{\Delta\omega}{u_0}\theta_2\sin\phi - k_0\theta_2^2\cos\phi - 2k_0\theta_2\sin\phi = 0\,. \tag{12}$$

An analysis of Eqs. (11) and (12) shows that linear angular dispersion of noncollinear interaction, Eq. (10), is obtained at $V = u_0\left(\cos\phi \mp \sqrt{k_0 g_0}u_0\sin\phi\right)^{-1}$.

Thus, two different modes of X waves can be excited in OPG pumped by Bessel beam. The frequency $\omega_0 + \Delta\omega$ of the first X wave mode increases with an angle θ ($\theta = \sqrt{g_0/k_0}\Delta\omega$, positive angular dispersion), and the velocity of this mode is

$$V_1 = u_0\left(\cos\phi - \sqrt{k_0 g_0}u_0\sin\phi\right)^{-1}\,. \tag{13}$$

The frequency of the second mode decreases with an angle θ (negative angular dispersion), and the velocity V_2 is

$$V_2 = u_0\left(\cos\phi + \sqrt{k_0 g_0}u_0\sin\phi\right)^{-1}\,. \tag{14}$$

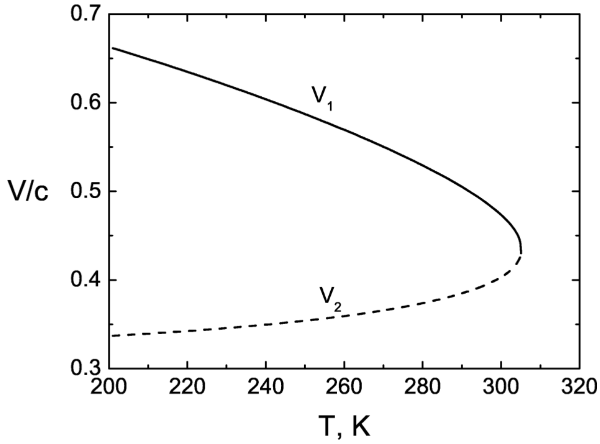

Fig. 3. Dependences of the velocities of two X wave modes on temperature in $LiNbO_3$ crystal. $\lambda_3 = 0.532$ μm.

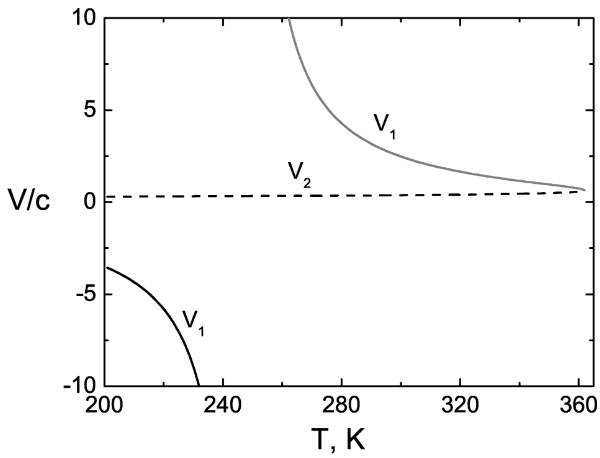

Fig. 4. Dependences of the velocities of two X wave modes on temperature in KDP crystal. $\lambda_3 = 0.266$ μm.

The temperature dependences of the velocities V_1 and V_2 for $LiNb0_3$ and KDP crystals are presented in Figs. 3 and 4, respectively. The first mode with velocity V_1 is superluminal ($V_1 > u_0$), while the other one with velocity V_2 ($V_2 < u_0$) is subluminal. We note that in KDP crystal at $T < 240$ K Bessel beam pump supports an existence of X wave propagating in opposite direction.

In conclusion, it has been shown that in OPG pumped by Bessel beam there can exist two different X wave modes. The mode with positive linear angular dispersion is superluminal, and the direction of its velocity can be opposite to the velocity of Bessel beam. Another X wave mode with negative linear angular dispersion is subluminal. We note that velocities of X wave modes can be varied by temperature of nonlinear crystal in conjunction with a variation of pump beam cone angle 2ϕ.

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PARAMETRINĖ ŠVIESOS X BANGŲ GENERACIJA, KAUPINANT BESELIO PLUOŠTU

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Santrauka

Parodyta, kad, kaupinant parametrinį generatorių šviesos Beselio pluoštu, jame gali būti sužadintos dvi skirtingos X bangų modos, kurioms būdingi skirtingi sklidimo greičiai. Tam tikrais atvejais yra įmanoma priešpriešinė vienos iš X modų ir Beselio pluošto parametrinė sąveika.