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SELF-ACTION OF THE PULSED CONICAL LIGHT BEAM IN NONLINEAR MEDIUM

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The self-action of the pulsed conical beam in the nonlinear medium is theoretically analysed. The results of the numerical calculation of nonlinear Schrödinger equation are presented. The generation of on-axis beam with broad frequency spectrum is foreseen and confirmed by the experimental observation in water.

Keywords: Bessel beam, nonlinear optics, self-action

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1. Introduction

The monochromatic conical beam is a superposition of plane waves of the same frequency with wave vectors that lie upon the surface of a cone. The spatial spectrum of a conical beam is ring-like. In general, when complex amplitudes of constituting plane waves are uncorrelated the conical beam is incoherent. If the amplitudes are equal, the coherent conical beam is the zeroth-order Bessel beam.

In the last years there has been much activity in the applications of Bessel beams in nonlinear optics because these beams can produce a strongly peaked intensity distribution over long spans in nonlinear media. Firstly, a self-action of Bessel beam was studied in gases at breakdown intensities [1, 2], and the longitudinal self-modulation of propagating beam intensity was observed. The distortion of angular spectrum of a Bessel beam after passing through the liquids was observed [3]. The angular spectrum of the beam showed coaxial ring structure. The phenomenon was attributed to phase aberrations induced by the intensity dependent refractive index. The appearance of the central spot (on-axis beam) in an angular spectrum of intense Bessel beam passed through benzene was observed [4]. This modification of an angular spectrum was attributed to self-action of Bessel beam in the medium with cubic nonlinearity under strong influence of diffraction. The nonlinear self-reconstruction of the truncated in azimuth Bessel beam has been observed [5, 6]. The nonlinear dynamics of Bessel–Gauss beams in a Kerr medium described by the nonlinear Schrödinger equation was investigated analytically and numerically [7], and, more recently, experimentally [8]. It was shown that the input Bessel beam experiences strong nonlinear reshaping due to the combined action of self-focusing and nonlinear losses. The modifications of an angular spectrum of the intense Bessel J_0 and J_1 beams caused by self-action in a thin colour glass plate with large cubic nonlinearity were investigated [9]. The appearance of outer ring of triple radius was observed. The phenomenon was explained as Bragg diffraction of Bessel beam on Bessel lattice in nonlinear medium under condition of negligible diffraction.

In general, an appearance of on-axis beam is a characteristic feature of self-acting Bessel beam in focusing medium. In what follows, we demonstrate that the on-axis beam with broad frequency spectrum can be produced due to self-action of pulsed conical beam in the nonlinear medium.

2. The model equation

The self-action of pulsed light beam in the dispersive nonlinear medium can be described by the nonlinear Schrödinger equation:

$$\frac{\partial A}{\partial z} - \frac{\mathrm{i}}{2k}\Delta_\perp A + \mathrm{i}\,\frac{g}{2}\,\frac{\partial^2 A}{\partial t^2} - \mathrm{i}\,\sigma|A|^2 A + \gamma|A|^4 A = 0\,, \tag{1}$$

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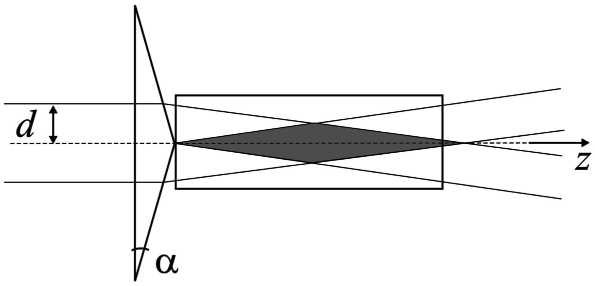

Fig. 1. Schematic depiction of the propagation of the Gaussian pulsed beam focused by axicon in the nonlinear medium.

where A is a complex amplitude of the light electric field, $\Delta_\perp = 1/r\,\partial/\partial r + \partial^2/\partial r^2$, where r and z are radial and longitudinal coordinates, respectively. k is a wave vector, t is time, and g is the group velocity dispersion (GVD) coefficient. $\sigma = n_0 n_2 \pi/(\lambda_0 \eta_0)$ is a nonlinear parameter, where n_0 is an index of the linear medium and n_2 is nonlinear index coefficient. Here λ_0 is the central wavelength and η_0 = 377 Ω is the wave resistance in the free space. We assume that nonlinear losses can be significant. In this case, γ is a coefficient of three-photon absorption. Further we consider the case of the self-focusing medium ($n_2 > 0$) with a normal GVD ($g > 0$). We suppose that the pulsed Gaussian beam with an amplitude

$$A_0 = a_0 \exp\left[-\left(\frac{r}{d}\right)^2 - \left(\frac{t}{\tau_0}\right)^2\right] \tag{2}$$

is focused by an axicon and afterwards propagates in the nonlinear medium (water, as in Fig. 1). Here d is the beam radius and τ_0 is the pulse duration. Then, the boundary condition for Eq. (1) is

$$A(r,t,z=0) = A_0 \exp(-\mathrm{i}\beta_0 r)\,, \tag{3}$$

where $\beta_0 \simeq 2\pi(n_\mathrm{a}-1)\alpha/\lambda_0$. Here α is the base angle of the thin glass axicon, n_a is glass refractive index, and the dispersion of β_0 is neglected. In the free space an axicon produces the quasi-Bessel beam [10]. The central part of it, close to beam axis, is similar to the pattern of an ideal Bessel beam. In the peripheral part the intensity oscillations are absent.

Here we investigate the focusing of pulsed Gaussian beam by axicon in the nonlinear medium. We note that just behind the axicon a Gaussian beam with a conical wavefront is obtained. In fact, this beam is a conical beam with a ring-like spatial spectrum.

Further, we introduce the dimensionless variables $\rho = \beta_0 r$, $\tau = t/\tau_0$, $\xi = z/L_p$, where $L_p = 2k/\beta_0^2$, and provide normalization of an amplitude $A = Ba_0$. Then, Eq. (1) can be rewritten in the form

$$\frac{\partial B}{\partial \xi} - \mathrm{i}\,\Delta_{\perp n} B + \mathrm{i}\,\frac{L_p}{L_\tau}\,\frac{\partial^2 B}{\partial \tau^2} - \mathrm{i}\,\frac{L_p}{L_\sigma}\,|B|^2 B + \frac{L_p}{L_\gamma}\,|B|^4 B = 0\,, \tag{4}$$

where $\Delta_{\perp n} = 1/\rho \cdot \partial/\partial\rho + \partial^2/\partial\rho^2$, $L_\tau = 2\tau_0^2/g$, $L_\sigma = 1/(\sigma a_0^2)$, and $L_\gamma = 1/(\gamma a_0^4)$. The boundary condition of Eq. (4) at $\xi = 0$ is

$$B(\rho,\tau,0) = \exp\left[-\left(\frac{\rho}{m}\right)^2 - \tau^2\right] \exp(-\mathrm{i}\rho)\,, \tag{5}$$

where $m = \beta_0 d$. Usually for beams focused by axicon we have $m \gg 1$.

The spectrum F of spatial β and temporal Ω frequencies of the wave with the amplitude A is given by

$$F(\beta,\Omega,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(r,t,z)\exp(\mathrm{i}\Omega t)\,J_0(r\beta)\,r\,\mathrm{d}r\,\mathrm{d}t\,. \tag{6}$$

Here $\Omega = \omega - \omega_0$, where ω is a frequency, $\omega_0 = 2\pi c/\lambda_0$ is a central frequency, and c is the velocity of light. Introducing the normalized variables $p = \beta/\beta_0$ and $f = \Omega\tau_0$ one obtains that the normalized angular spectrum $S(p,f,\xi)$ can be written as

$$S(p,f,\xi) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} B(\rho,\tau,\xi)\exp(\mathrm{i}f\tau)\,J_0(p\rho)\,\rho\,\mathrm{d}\rho\,\mathrm{d}\tau\,, \tag{7}$$

where $S(p,f,\xi) = (\tau_0 a_0)/\beta_0^2 \cdot F(\beta,\Omega,z)$.

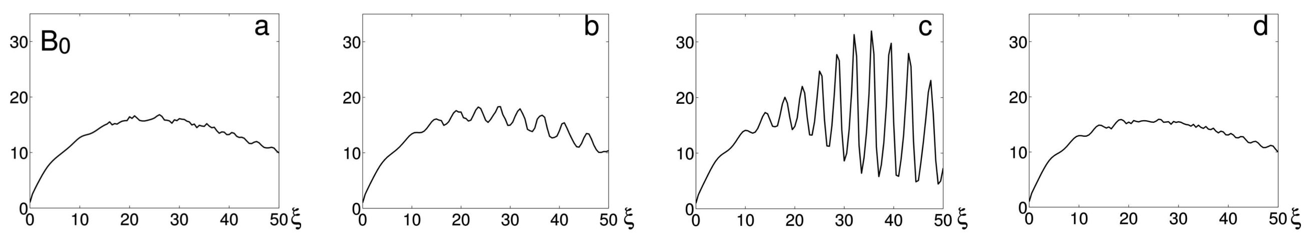

Fig. 2. Dependence of normalized amplitude B_0 on propagation distance ξ, when (a–c) $L_p/L_\gamma = 10^{-6}$, (d) $L_p/L_\gamma = 10^{-5}$. L_p/L_σ: (a) 0.005, (b) 0.0075, (c, d) 0.01.

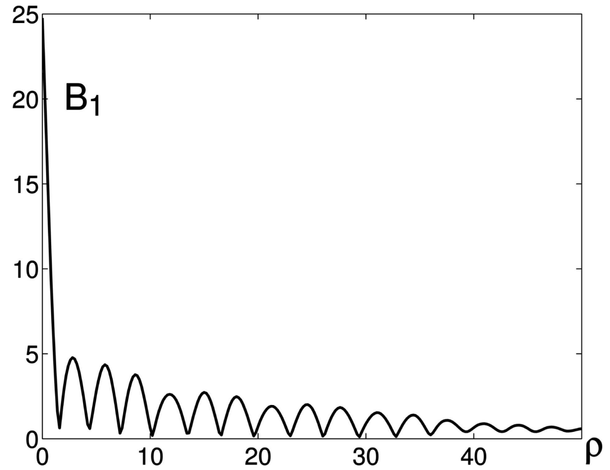

Fig. 3. Radial distribution of the beam amplitude $B_1 = |B(\rho, \tau = 0)|$ in the nonlinear medium at $\xi = 25$. $L_p/L_\gamma = 10^{-6}$, $L_p/L_\sigma = 0.01$.

3. Computer simulation of the model equation

The results of numerical simulation of Eq. (4) as well as numerically calculated angular spectrum $|S|/|S_0|$ are depicted in Figs. 2–8. Here $|S_0|$ is a peak value at $\xi = 0$. The values $m = 100$ and $L_p/L_\tau = 0.001$ were taken. When the parameters L_p/L_σ and L_p/L_γ are small, the typical dependence of an amplitude along the beam axis $B_0 = |B(0, 0, \xi)|$ on the propagation distance (Fig. 2(a)) is the same as that for linear medium, see [10]. The oscillations of B_0 arise, when the values of the parameter L_p/L_σ are larger but the parameter L_p/L_γ remains small (Fig. 2(b, c)). The oscillations disappear if the parameter of nonlinear losses L_p/L_γ is increased (Fig. 2(d)). In general, the nonlinear quasi-Bessel beam is formed (Fig. 3).

An evolution of the spatial-temporal spectrum $|S(p, f, \xi)|$ is depicted in Fig. 4. The dimensionless variable $f_0 = \omega_0 \tau_0 = 710$ was involved that corresponds to $\lambda_0 = 532$ nm and $\tau_0 = 200$ fs. The central spot at $p = 0$ as well as outer rings at $p > 1$ are observed due to self-action of conical beam in the nonlinear medium. It can be seen that the spectrum of on-axis beam ($p = 0$) at $\xi = 40$ (Fig. 4(d)) is much broader than the spectrum

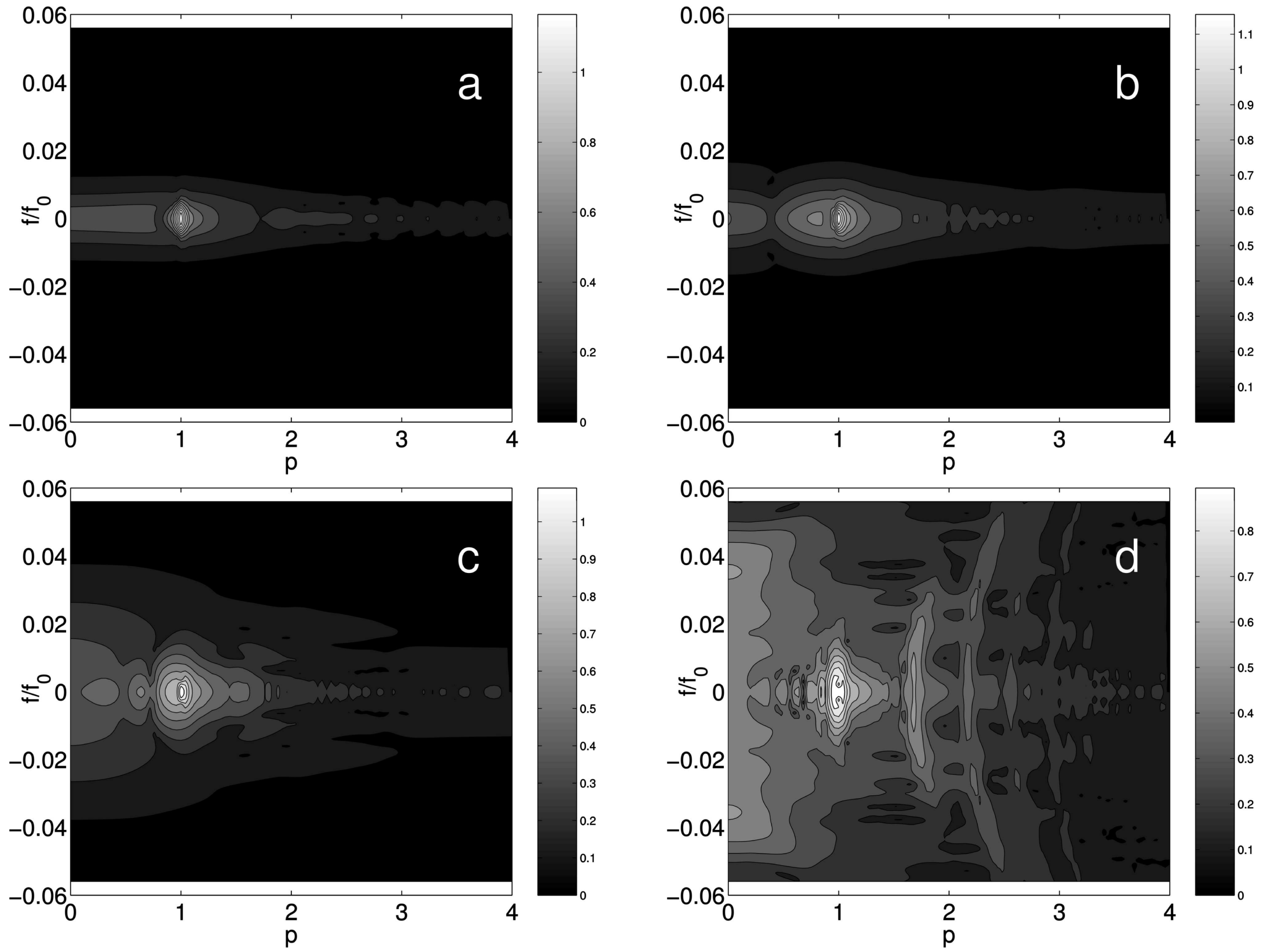

Fig. 4. Evolution of spatial-temporal spectrum $|S(p, f, \xi)|^{2/3}$ of pulsed conical beam under self-action in nonlinear medium. $L_p/L_\gamma = 10^{-6}$, $L_p/L_\sigma = 0.01$. ξ: (a) 5, (b) 10, (c) 20, (d) 40.

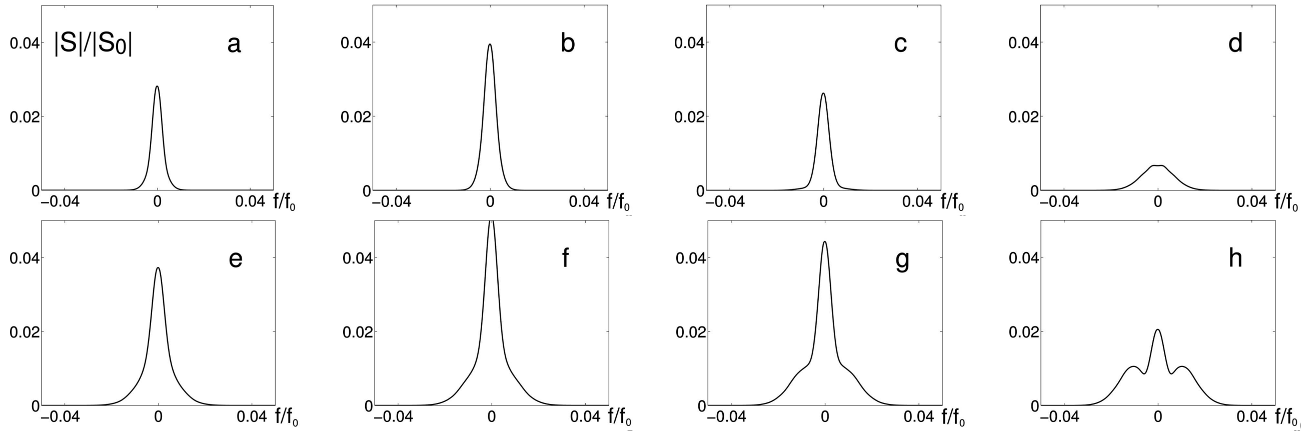

Fig. 5. Temporal spectrum $|S(p = 0, f, \xi)|/|S_0|$ of on-axis beam under self-action of pulsed conical beam in nonlinear medium. $L_p/L_\gamma = 10^{-6}$, $L_p/L_\sigma = 0.005$. ξ: (a) 5, (b) 10, (c) 15, (d) 20, (e) 25, (f) 30, (g) 35, (h) 40.

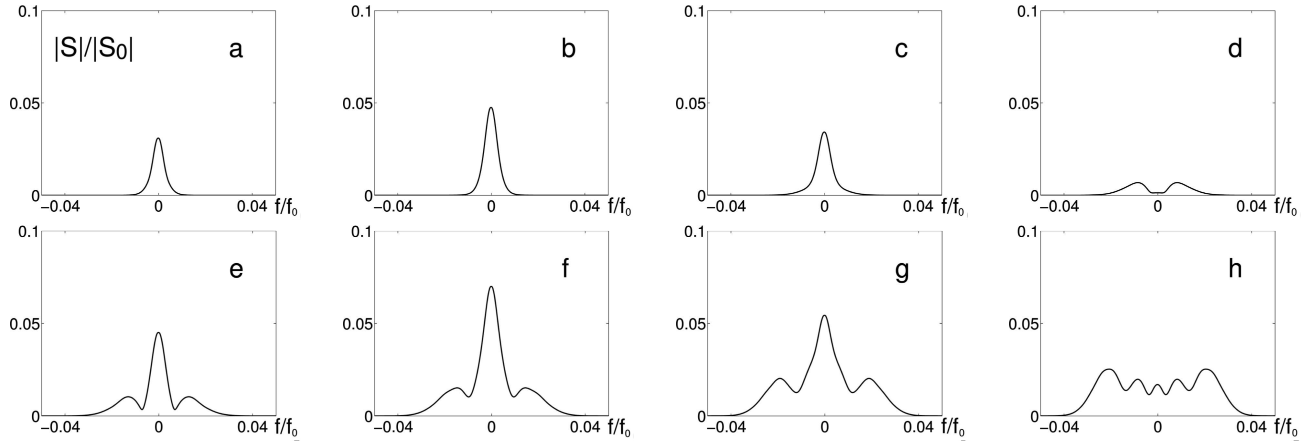

Fig. 6. Temporal spectrum $|S(p = 0, f, \xi)|/|S_0|$ of on-axis beam under self-action of pulsed conical beam in nonlinear medium. $L_p/L_\gamma = 10^{-6}$, $L_p/L_\sigma = 0.0075$. ξ: (a) 5, (b) 10, (c) 15, (d) 20, (e) 25, (f) 30, (g) 35, (h) 40.

at $\xi = 5$ and $p = 1$ (Fig. 4(a)). The evolution of on-axis spectrum $S(p = 0, f, \xi)$ for various values of L_p/L_σ and L_p/L_γ is presented in Figs. 5–8. The bandwidth of the temporal frequencies increases with the increase of the parameter L_p/L_σ and with the decrease of the parameter of nonlinear losses L_p/L_γ. We note that at $L_p/L_\sigma \geq 0.02$ and $L_p/L_\gamma = 10^{-6}$ a self-focusing of conical beam takes place. In general, the on-axis beam with broad frequency spectrum (light continuum) can be produced by self-action of conical beam in nonlinear medium.

4. Discussion and conclusions

We shall present the conditions needed for an appearance of on-axis beam with broad frequency spectrum due to self-action of conical beam in a water cell (Fig. 1).

For the Gaussian beam diameter we choose 0.8 mm, pulse duration 200 fs, central wavelength $\lambda_0 = 532$ nm, and beam power $P = 10P_{\mathrm{cr}}$, where $P_{\mathrm{cr}} = 1.15$ MW is the critical power for continous wave (cw) beam collapse in water. In this case for beam intensity we obtain $I = 4.5$ GW/cm^2. For glass axicon we take the base angle $\alpha_0 = 40$ mrad and the refractive index $n_{\mathrm{a}} = 1.52$. Then we find $m \approx 100$ and $L_p \approx 0.5$ mm. Assuming for water the refractive index $n_0 = 1.33$, nonlinear refractive index $n_2 = 2.7\cdot10^{-16}$ cm^2/W [11], GVD coefficient $g_0 = 0.056$ fs$^2/\mu$m at λ_0 [12], and three-photon absorption coefficient $\gamma \approx 10^{-24}$ cm^3/W^2 [13] we find: $L_\sigma \approx 5.7$ cm, $L_\tau = 36$ cm, and $L_\gamma \approx 330$ m. At these values, the parameters of Eq. (4) are: $L_p/L_\tau = 1.38\cdot10^{-3}$, $L_p/L_\sigma = 8.8\cdot10^{-3}$, and $L_p/L_\gamma = 1.5\cdot10^{-6}$. The values of the parameters calculated for water are in good agreement with the conditions needed for the significant broadening of on-axis beam spectrum in nonlinear medium due to self-action of conical beam produced by axicon (Figs. 5–7).

The results of numerical simulations were verified experimentally, by launching 200 fs, 527 nm Bessel

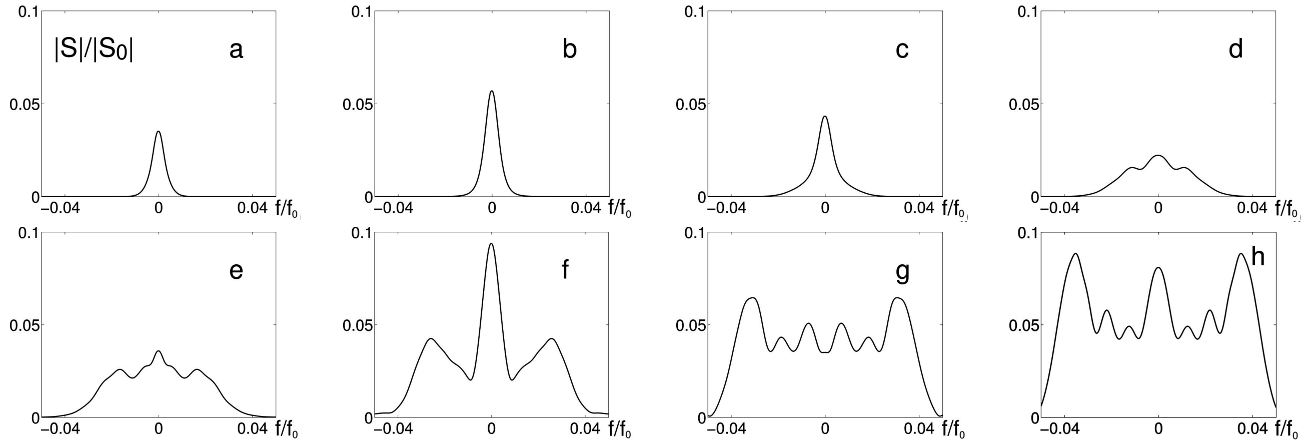

Fig. 7. Temporal spectrum $|S(p = 0, f, \xi)|/|S_0|$ of on-axis beam under self-action of pulsed conical beam in nonlinear medium. $L_p/L_\gamma = 10^{-6}$, $L_p/L_\sigma = 0.01$. ξ: (a) 5, (b) 10, (c) 15, (d) 20, (e) 25, (f) 30, (g) 35, (h) 40.

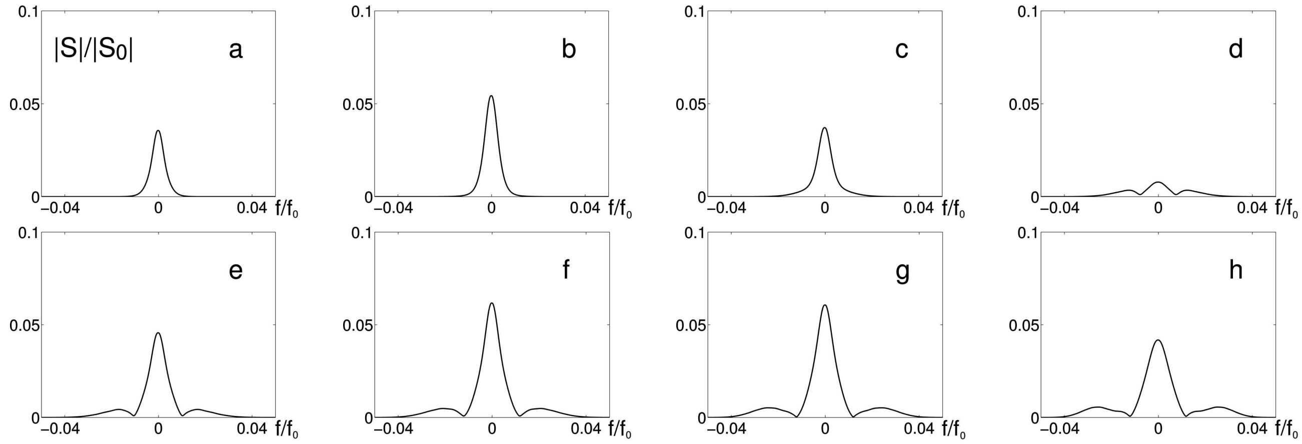

Fig. 8. Temporal spectrum $|S(p = 0, f, \xi)|/|S_0|$ of on-axis beam under self-action of pulsed conical beam in nonlinear medium. $L_p/L_\gamma = 10^{-5}$, $L_p/L_\sigma = 0.01$. ξ: (a) 5, (b) 10, (c) 15, (d) 20, (e) 25, (f) 30, (g) 35, (h) 40.

beam into 40 mm long water-filled cell, as shown in Fig. 1. The ultrashort pulsed Bessel beam was generated by means of BK7 glass axicon of $2°50'$ base angle. The axicon-generated Bessel beam had a Bessel zone of about 50 mm length and a width of the central maximum of 7 μm (FWHM). For input energy above 10 μJ, nonlinear propagation through the water cell resulted in modified spatial spectrum, as measured in the Fourier plane of the f = 50 mm achromatic lens, see Fig. 9(a). Typical frequency spectra of the generated axial radiation for various input energies as recorded by fibre spectrometer are shown in Fig. 9(b–d). Experimental data are in good qualitative agreement with the simulation results.

In conclusion, we have predicted and measured a noticeable spectral broadening of an axial component generated in the femtosecond Bessel beam after propagation in nonlinear medium (water), which occurs as a result of strong self-action effect.

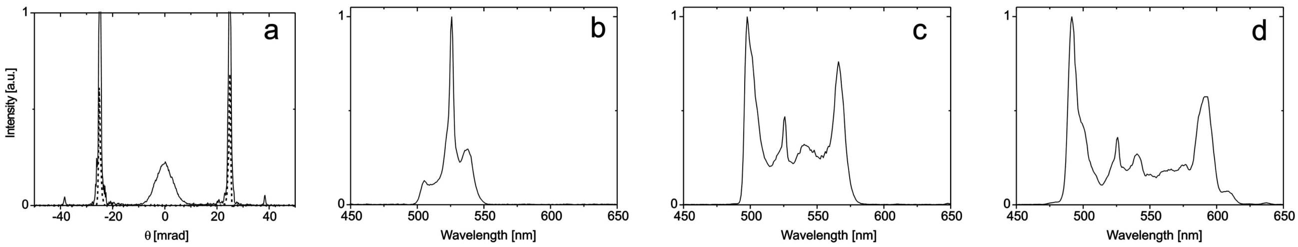

Fig. 9. Experimental data: (a) spatial spectrum of the input Bessel beam (shown by dashed line) after propagation of 40 mm in water; (b–d) temporal spectra of an axial radiation for three different input energies, (b) 12.3 μJ, (c) 14.5 μJ, (d) 15.3 μJ.

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IMPULSINIO KŪGINIO ŠVIESOS PLUOŠTO SAVIVEIKA NETIESINĖJE TERPĖJE

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Santrauka

Teoriškai išnagrinėta impulsinio kūginio pluošto saviveika netiesinėje terpėje. Pateikti netiesinės Šrėdingerio (Schrödinger) lygties skaitmeninio modeliavimo rezultatai. Numatytas ir eksperimentiškai pademonstruotas plačios dažnių juostos ašinio pluošto susidarymas vandenyje.