STABILIZATION OF UNSTABLE PERIODIC ORBIT IN CHAOTIC DUFFING–HOLMES OSCILLATOR BY SECOND ORDER RESONANT NEGATIVE FEEDBACK [*]

A. Tamaševičius [a], E. Tamaševičiūtė [a,b], G. Mykolaitis [a], and S. Bumelienė [a]

[a] *Semiconductor Physics Institute, A. Goštauto 11, LT-01108 Vilnius, Lithuania*
E-mail: tamasev@pfi.lt
[b] *Department of Radiophysics, Vilnius University, Saulėtekio 9, LT-10222 Vilnius, Lithuania*

We consider a second order linear resonator inserted in the negative feedback loop of the chaotic Duffing–Holmes oscillator for stabilizing unstable periodic orbit. Mathematical model is discussed and numerical simulations are presented. An analogue electronic controller is described. Experiments have been performed with an electronic version of the Duffing–Holmes oscillator. Stabilization of periodic oscillations can be achieved with a small control force.

Keywords: nonlinear dynamics, low-dimensional chaos, control of chaos

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1. Introduction

17 years ago Edward Ott, Celso Grebogi, and James Yorke launched a new and very intriguing activity in nonlinear dynamics, called 'Controlling Chaos' [1], still attractive nowadays. It inspired many other researchers to develop alternative control algorithms [2–4]. One of the most successful techniques is the time-continuous delayed feedback control (DFC) method, also known as Pyragas' method [5–7]. Practical implementation of the DFC method requires a delay line in the feedback circuit. At high frequencies (hundreds of megahertz) the delay line is either simply a coaxial transmission cable or a micro-strip line. However, at lower frequencies the delay unit is rather complicated and inconvenient device. In the present paper we suggest an extremely simple technique for controlling unstable periodic orbits embedded in chaotic systems. The basic idea behind the method is to replace the feedback loop containing a delay line with the second order resonant negative feedback (RNF2) where the resonator is tuned to the main frequency of the unstable orbit. As a result the negative feedback damps all the oscillations except the desired periodic orbit. To be specific we consider the chaotic Duffing–

Holmes oscillator [8–10] and employ a second order LC filter similarly to the very recently described technique of synchronization of one-way coupled simple periodic oscillators [11].

2. Mathematical model and numerical results

The Duffing–Holmes two-well oscillator [8–10] is given either by the second order nonlinear equation

$$\ddot{x} + b\dot{x} - x + x^3 = a \sin \omega t \tag{1}$$

or by the set of two first order equations

$$\dot{x} = y \,,$$
$$\dot{y} = -by + x - x^3 + a \sin \omega t \,. \tag{2}$$

In Eqs. (1), (2) b is the damping coefficient, a and ω is the amplitude and frequency of external periodic force, respectively. Characteristic frequency of the nonlinear oscillator, applicable in the vicinity of the unforced ($a = 0$) stable steady states $x_0 = \pm 1$, i.e. close to the bottoms of the wells, is $\omega_0 = 2^{1/2}$.

Let us couple the nonlinear Duffing–Holmes oscillator with a damped second order linear oscillator, de-
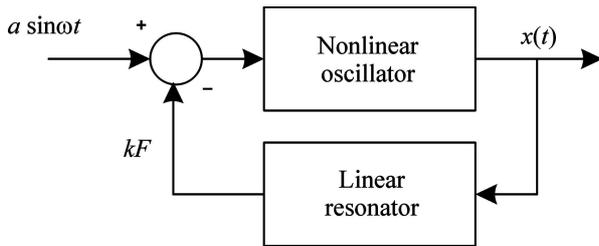
Fig. 1. Block diagram of the control technique.

Fig. 3. Waveforms from Eq. (3). Parameters are the same as in Fig. 2. Control is activated at the time moment $t = 300$. The lower trace is intentionally shifted down by value -3.
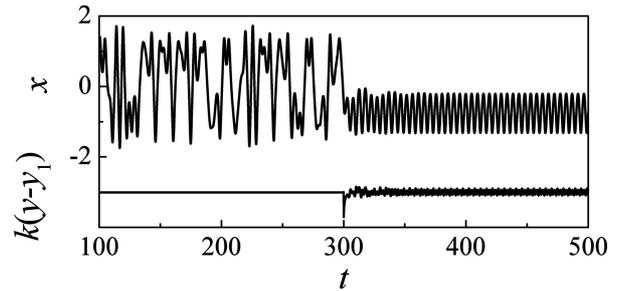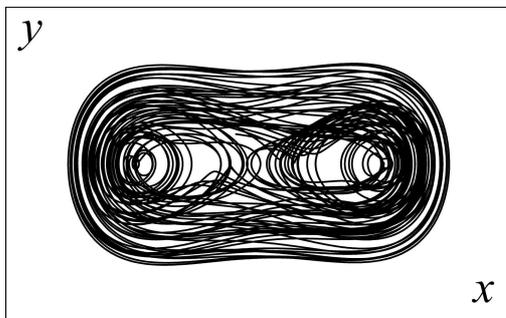
scribed by dynamical variables x_1, y_1, damping parameter b_1, and characteristic frequency ω_1:

$$\dot{x} = y\,,$$

$$\dot{y} = -by + x - x^3 + a\,\sin\omega t - k(y - y_1)\,,$$

$$\dot{x}_1 = \omega_1(y_1 - b_1 x_1)\,,$$

$$\dot{y}_1 = \omega_1[-x_1 + k_1(y - y_1)]\,. \tag{3}$$

Here the coupling coefficients k and k_1 may be different. Equations (3) can be presented in the following form:

$$\ddot{x} + b\dot{x} - x + x^3 = a\,\sin\,\omega t - k\,F(\dot{x}, \dot{x}_1, x_1)\,,$$

$$\ddot{x}_1 + \omega_1 b_1 \dot{x}_1 + \omega_1^2 x_1 = \omega_1^2 k_1 F(\dot{x}, \dot{x}_1, x_1)\,, \tag{4}$$

where the control feedback term F is given by

$$F(\dot{x}, \dot{x}_1, x_1) = \dot{x} - \frac{1}{\omega_1}\dot{x}_1 - b_1\,x_1\,. \tag{5}$$

Control set-up containing linear resonator in the negative feedback loop of the nonlinear oscillator is sketched in Fig. 1. Numerical results obtained from Eq. (3) using the simulation program *Mathematica* are shown in Figs. 2 and 3.

3. Experimental set-up

The suggested control technique has been tested experimentally using the set-up depicted in Fig. 4. The subcircuit composed of the operational amplifier OA1,
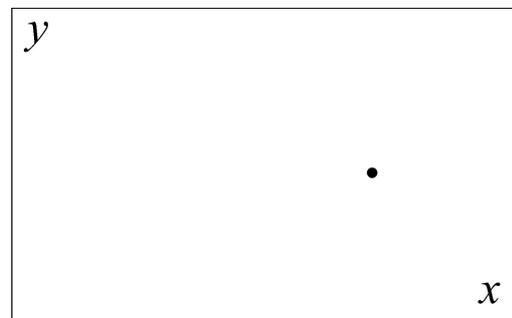

(a)
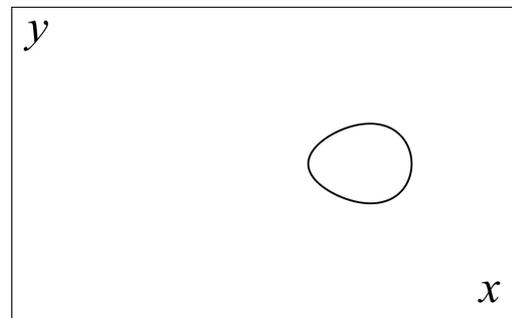
(b)

Fig. 2. Simulated results from Eq. (3). (a) Phase portraits, (b) Poincaré sections: (left) without control, $k = k_1 = 0$, (right) under control, $k = 2$, $k_1 = 1$. Other parameter values: $a = 0.45$, $b = 0.1$, $b_1 = 0.01$, $\omega = 1.3$, $\omega_1 = \omega$.
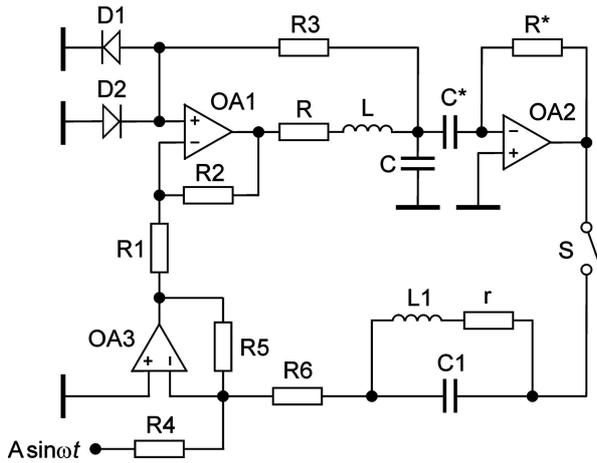
Fig. 4. Circuit diagram of the Duffing–Holmes oscillator with a controller in the feedback loop. $\omega = 2\pi f$, $L = 19$ mH, $C = 470$ nF ($f_0 \approx 1.7$ kHz), $L_1 = 47$ mH, $C_1 = 240$ nF ($f_1 \approx 1.5$ kHz), $R = 20\,\Omega$, $r = 51\,\Omega$, $R_1 = R_2 = R_3 = 10$ kΩ, $R_4 = R_5 = 1$ kΩ, $R_6 = 510\,\Omega$. Here S is an electronic switch.

the R–L–C resonator, the resistors R1–R3 and the diodes D1, D2 is an electronic implementation of the nonautonomous Duffing–Holmes oscillator described in detail elsewhere [12]. It is used in [13] and is similar in a sense to the Young–Silva circuit [14], but is essentially simpler. The rest of the circuit is the controller. The basic element is the resonant filter r–L1–C1. The OA2 based stage is a differentiator and the OA3 stage is an adder. The input signal of the differentiator is the voltage across the capacitor C, $V_{\text{in}} = V_C$, the output signal $V_{\text{out}} = -R^* \cdot C^* \cdot \dot{V}_C \propto -\dot{x} = -y$. The val-

ues of R^* and C^* are set so that $R^*C^* = (LC)^{1/2}$. The control coefficients k and k_1 are related to the circuit element values as follows: $k = R_5/R_6$, $k_1 = (L_1/C_1)^{1/2}/R_6$.

4. Experimental results

The photos demonstrating a settled behaviour of the free-running oscillator (switch S is in the opened position) and the controlled oscillator (S is closed) are shown in Fig. 5. Closed loop and a single dot in the right-hand plots of the phase portraits and the Poincaré sections, respectively, confirm that the RNF2 method successfully stabilizes period-1 orbit. A snapshot of the waveform including transient process is presented in Fig. 6. The experimental results are in a good agreement with the simulated plots in Figs. 2, 3.

5. Concluding remarks

Stabilization of the periodic orbit in the Duffing–Holmes oscillator has been already demonstrated in earlier papers both numerically [5] and experimentally [15] using the DFC method. As mentioned in the Introduction the RNF2 method has been recently successfully applied to extend the region of synchronization of two simple periodic oscillators [11]. In the present paper we have described a similar RNF2 analogue controller and have applied it to stabilize unstable
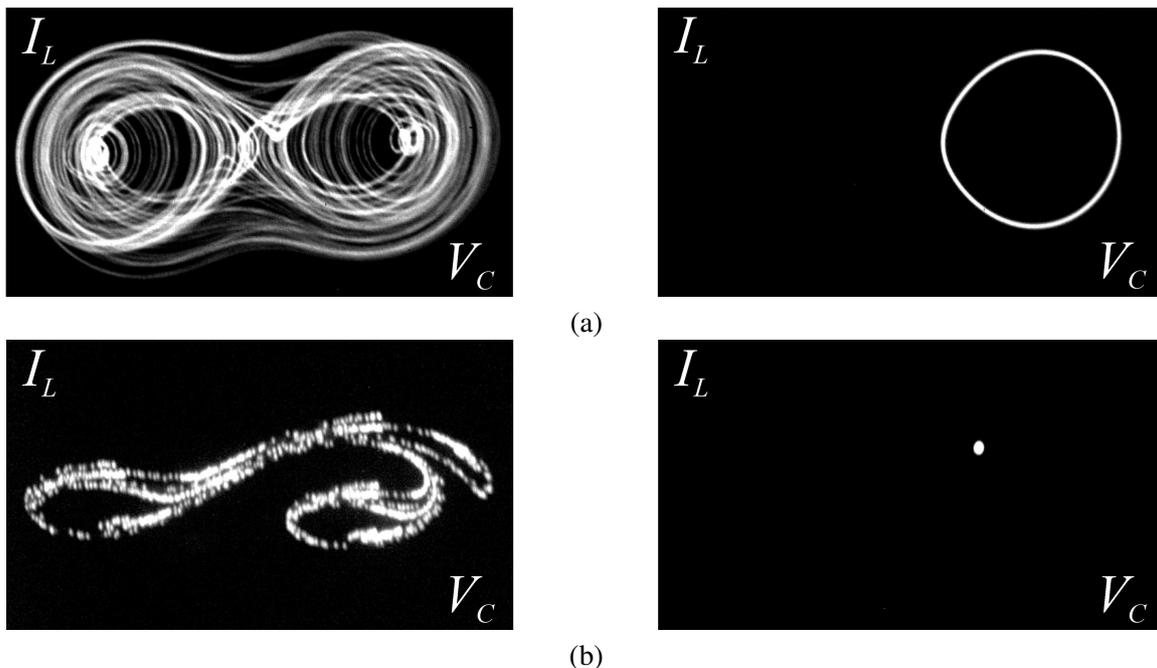

(a)

(b)

Fig. 5. Experimental plots. (a) Phase portraits, (b) Poincaré sections: (left) without control, (right) under control. $A = 200$ mV, $f = 1.5$ kHz, $f_1 = f = 1.5$ kHz.
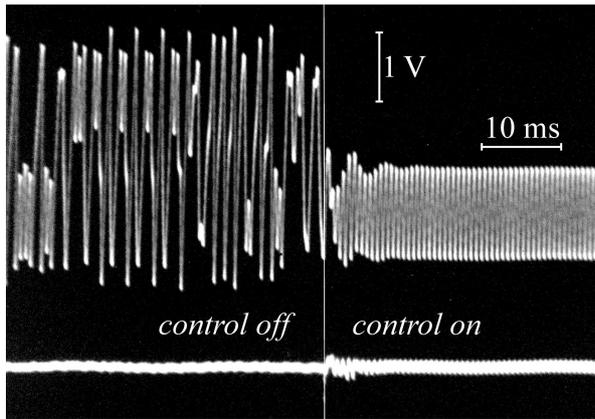
Fig. 6. Experimental waveforms. Upper trace is the output signal $V_C(t)$ and lower trace is the control signal $k\,F(t)$, taken across resistor R6. Parameters are the same as in Fig. 5. Fine vertical line in the photo divides the regions where control is off (S opened) and control is on (S closed).

periodic orbit in a chaotic electronic circuit that mimics the Duffing–Holmes oscillator [12]. In contrast to the DFC method the residual control signal in the RNF2 method does not vanish. However, it appears to be sufficiently small (about 10% compared to the main signal). In the RNF2 method only the first harmonic of the stabilized periodic orbit remains unchanged, but its higher harmonics may be slightly affected. Detailed numerical and experimental analysis (not discussed in the previous sections) shows that the main component of the residual control force is just the second harmonic. Therefore the RNF2 controller can be easily improved by inserting in the feedback loop an additional second order resonator with the resonance frequency twice higher as that of the main resonator.

We would like to make some remarks concerning control of chaos via so-called dynamical vibration absorber (DVA) [2, 16]. The DVA method is similar in a sense to the RNF2 method. A linear "physically small" [2] oscillator (a resonant absorber) is coupled to the main chaotic oscillator. It can change the overall dynamics, thus suppressing chaos and stabilizing periodic orbits. The DVA method has been tested numerically [2, 16] by applying it to the Duffing–Ueda oscillator [17], which is similar to the Duffing– Holmes oscillator, but without the term '$-x$' in Eq. (1) and therefore has a simpler single-well potential. The control force in the DVA method is different from that of the RNF2 method. It exploits the difference of coordinates $F \propto (x - x_1)$, instead of the difference of their derivatives, e.g. F given by Eq. (5). Finally, Kapitaniak [2, 16] misleadingly attributes the DVA method to chaos control methods "without feedback".

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NESTABILIOS PERIODINĖS ORBITOS STABILIZAVIMAS CHAOTINIAME DUFFING'O IR HOLMES'O OSCILIATORIUJE ANTROS EILĖS REZONANSINIU NEIGIAMU GRĮŽTAMUOJU RYŠIU

A. Tamaševičius [a], E. Tamaševičiūtė [a,b], G. Mykolaitis [a], S. Bumelienė [a]

[a] *Puslaidininkių fizikos institutas, Vilnius, Lietuva*
[b] *Vilniaus universitetas, Vilnius, Lietuva*

Santrauka

Aprašėme labai paprastą nestabilių periodinių orbitų, esančių chaotiniuose atraktoriuose, stabilizavimo būdą, paremtą antros eilės rezonansiniu neigiamu grįžtamuoju ryšiu (RNGR2). Skirtingai nuo Pyrago uždelstojo grįžtamojo ryšio (UGR) metodo, RNGR2 metode panaudojama ne vėlinimo linija, bet antros eilės rezonatorius. Toks metodas jau anksčiau buvo pritaikytas periodinių virpesių generatorių sinchronizacijos riboms išplėsti. Panašiai kaip ir UGR metode, naudojant RNGR2 metodą nereikia žinoti valdomosios sistemos modelio, tačiau eksperimentiškai realizuoti RNGR2 valdiklį yra žymiai lengviau nei UGR valdiklį.

Šiame darbe išnagrinėjome išorinės periodinės jėgos veikiamą netiesinį Duffing'o ir Holmes'o osciliatorių, kuris plačiame parametrų ruože elgiasi chaotiškai. Pateikėme diferencialinių lygčių skaitinius sprendinius. Metodą išbandėme ir eksperimentiškai. Šiam tikslui sukūrėme analoginį RNGR2 valdiklį. Jo pagrindinis elementas yra antros eilės LC virpesių kontūras, kurio rezonansinis dažnis atitinka išorinės periodinės jėgos dažnį. RNGR2 valdiklį pritaikėme elektroninio Duffing'o ir Holmes'o osciliatoriaus valdymui. Prijungus valdiklį, po neilgo pereinamojo proceso osciliatorius pereina iš chaotinių virpesių į periodinių virpesių režimą. Periodinių virpesių dažnis šiuo atveju sutampa su išorinės jėgos dažniu, t. y. stabilizuojama pirmoji nestabili periodinė orbita. Naudojant RNGR2 metodą, valdančioji jėga nevirsta nuline kaip UGR metode, bet išlieka baigtinio dydžio (apie 10%, lyginant su osciliatoriaus išėjimo signalu). RNGR2 metodas nekeičia tik pirmosios periodinių virpesių harmonikos, bet gali keisti aukštesniąsias harmonikas. Skaitiniai ir eksperimento rezultatai rodo, kad valdančios jėgos sudėtyje vyrauja antrosios harmonikos sandas, todėl RNGR2 valdiklį nesunku patobulinti, įjungiant papildomą antros eilės rezonatorių, suderintą antrajai harmonikai, t. y. paverčiant jį RNGR4 valdikliu.