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NONLINEAR GAUGE-FIXING IN THE FeynArts PACKAGE

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Accurate calculations in the Standard Model (SM) that match the experimental accuracy require the generation and evaluation of Feynman diagrams up to one- or even two-loop order, where there can be hundreds of diagrams. Programs like FeynArts or GRACE can do this in an automated way. In order to check the reliability, one can test if the result is independent of the gauge-fixing parameters. Nonlinear gauge-fixing, as proposed by F. Boudjema and E. Chopin, introduces more parameters, so the test is more stringent. In this paper we present the implementation of nonlinear gauge-fixing into the model files for FeynArts and discuss the gauge invariance of diagrams that are generated by FeynArts with the use of our model files.

Keywords: nonlinear gauge-fixing, computer programming

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1. Introduction

The accepted theoretical model of elementary particles and their interactions, the Standard Model (SM), is very successful at explaining the experimental data available. It describes all the observed particles, leptons and hadrons, that interact by the electromagnetic, weak, and strong forces. The predictions of the SM have been tested in particle collider experiments up to an energy scale of around 200 GeV in precision measurements and up to an energy scale of around 900 GeV in proton–antiproton collisions [1]. However, it is clear that some new effects should occur at higher energies or higher luminosities. One such effect is the highly anticipated Higgs scalar particle that is predicted by the SM and has yet to be found experimentally.

Due to the increased accuracy in the experimental area, the theoretical calculations should become more accurate as well, even in extensions of the SM [2]. The calculation in the perturbative approach requires to sum over all amplitudes contributing to the measured process. There are already tools developed, that are capable of automatically generating and calculating Feynman diagrams at one-loop level in the SM and in the Minimal Supersymmetric Standard Model (MSSM) (like [3–5] and others). But together with the development of such tools there is also a need for ways to check the validity of the computed results, in order to use them with any reliability. One powerful tool to perform such checks arises from the procedure of gauge-fixing the theory, as the result has to be independent of the introduced gauge-fixing parameters. Naturally, the more gauge-fixing parameters we have available, the more stringent the test becomes. The nonlinear gauge-fixing of the Standard Model was presented in [6], and later fully implemented in GRACE [5]. In this work we want to implement the same class of nonlinear gauge-fixing in the FeynArts / FormCalc package. The advantage of this package is that it is open-source and freely distributable.

In Section 2 we introduce a general Yang–Mills (YM) Lagrangian with a spontaneously broken gauge symmetry. We use the Faddeev–Popov procedure to fix this gauge symmetry in Section 3 and discuss the most general linear gauge-fixing in Section 4. In Section 5 we implement the gauge-fixing in the SM and work out how to obtain the full Lagrangian from the choices of gauge-fixing. We describe the implementation in FeynArts in Section 6 and conclude with an outlook and acknowledgements.

2. Yang–Mills Lagrangian and symmetry breaking

We consider a gauge theory for a multiplet of real scalar fields ϕ_i, transforming as some representation R

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of the gauge group G. The infinitesimal gauge transformation for the scalar fields ϕ_i and the gauge fields A^a_μ is then given by (see [7])

$$\delta\phi_i(x) = -\alpha^a(x) T^a_{ij}\phi_j(x)\,, \tag{1}$$

$$\delta A^a_\mu(x) = \frac{1}{g_a}\partial_\mu\alpha^a(x) + f^{abc}A^b_\mu(x)\alpha^c(x)\,, \tag{2}$$

where $\alpha^a(x)$ are the infinitesimal parameters of the gauge transformation, T^a_{ij} are the generators of the real representation R, f^{abc} are the structure constants of the gauge group G with $[T^a, T^b] = f^{abc}T^c$ and g_a is the coupling constant, which can be chosen independently for each simple or $U(1)$ subgroup of G and hence has a as an index for the subgroup.

The covariant derivative $(D_\mu\phi)_i = \partial_\mu\phi_i + g_a A^a_\mu \times T^a_{ij}\phi_j$ and the field strength tensor $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_a f^{abc}A^b_\mu A^c_\nu$ give the invariant Lagrangian for this gauge theory

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \frac{1}{2}(D_\mu\phi_i)^2 - V(\phi)\,, \tag{3}$$

where $V(\phi)$ is the symmetry-breaking scalar potential.

So far the gauge fields are massless. The quantum fluctuations can only happen around the minimum value of the field ϕ, which acquires a vacuum expectation value (VEV), $v_i \equiv \langle\phi_i\rangle$, if we take the potential $V(\phi)$ to have its minimum defined by $V_i(\phi) := \partial V(\phi)/\partial\phi_i = 0$, away from $\phi = 0$.

In order to use perturbation theory, we have to analyse fields that have a zero vacuum expectation value, so we redefine $\phi_i(x) = v_i + \chi_i(x)$ and treat $\chi_i(x)$ as our field of interest.

Plugging the definitions into Eq. (3) yields the Lagrangian in terms of χ_i. Expanding it up to the quadratic terms in the fields we get:

$$\mathcal{L}_2(A,\chi) = -\frac{1}{2}A^a_\mu(-g^{\mu\nu}\partial^2 + \partial^\mu\partial^\nu)A^a_\nu + \frac{1}{2}(\partial_\mu\chi_i)^2 + F^a{}_i A^a_\mu\partial^\mu\chi_i + \frac{1}{2}F^a{}_i F^b{}_i A^a_\mu A^{\mu b} - \frac{1}{2}M_{ij}\chi_i\chi_j\,, \tag{4}$$

with constant matrices $F^a \equiv g_a T^a_{ij}v_j$, and $M_{ij} \equiv \partial^2 V(\phi)/(\partial\phi_i\partial\phi_j)|_{V_i=0}$, the coefficient of the quadratic term in the Taylor series expansion of the potential $V(\phi)$. Now the gauge bosons have acquired a mass matrix $F^a{}_i F^b{}_i$, and the scalar fields have the mass matrix M_{ij}. This is the famous Higgs mechanism.

3. Gauge-fixing with the Faddeev–Popov procedure

Path-integral quantization proceeds by analysing the quantity

$$Z = \int \mathcal{D}A\,\mathcal{D}\chi\,\exp\left[\mathrm{i}\int\mathcal{L}(A,\chi)\right]\,, \tag{5}$$

from which the propagators and interaction vertices can be read off [1]. However, a gauge symmetry has to be fixed, because otherwise the integral in Eq. (5) runs over many field configurations that are gauge-equivalent, instead of counting each physical configuration just once.

Gauge-fixing is usually done by the Faddeev–Popov procedure, as a result of which Z is expressed as (see [7, 8]):

$$Z = C\int\mathcal{D}A\,\mathcal{D}\chi\,\exp\left[\mathrm{i}\int d^4x\,(\mathcal{L}[A,\chi] + \mathcal{L}_{\mathrm{gf}}(G^a))\right] \times \det\left[\left.\frac{\delta G^a[A_\alpha,\chi_\alpha;x]}{\delta\alpha^b(y)}\right|_{\alpha=0}\right]\,. \tag{6}$$

$G^a[A,\chi;x]$ are arbitrary functions, constraining in the sense that the condition $G^a = 0$ serves as a constraint on the fields in Eq. (6). The new gauge-fixing Lagrangian term $\mathcal{L}_{\mathrm{gf}}$ can be any function of G^a, while C is an irrelevant normalization constant. The subscript α of a field means that the field is gauge-transformed with the gauge transformation parameters α.

The determinant $\det\left[\delta G^a(x)/\delta\alpha^b(y)\right] = D^{ab}(x,y)$ can be evaluated by the path integral

$$\det[D^{ab}(x,y)] = \int\mathcal{D}\bar{c}\,\mathcal{D}c\,\exp\left[\mathrm{i}\int \mathrm{d}^4x\,\mathrm{d}^4y\,\bar{c}^a(x)D^{ab}(x,y)c^b(y)\right]\,, \tag{7}$$

where $\bar{c}^a(x)$ and $c^b(y)$ are anticommuting scalar fields, called ghosts. Since the functional derivative $\delta G(x)/\delta\alpha(y)$ is local, i. e., it contains $\delta(x-y)$ as a factor, we can express Eq. (6) as

$$Z = C\int\mathcal{D}A\,\mathcal{D}\chi\,\mathcal{D}\bar{c}\,\mathcal{D}c\,\exp\left[\mathrm{i}\int\mathrm{d}^4x\,(\mathcal{L}[A,\chi] + \mathcal{L}_{\mathrm{gf}}(G^a) + \mathcal{L}_{\mathrm{gh}}[\bar{c},c,A,\chi])\right]\,, \tag{8}$$

where

$$\mathcal{L}_{\mathrm{gh}} \equiv \bar{c}^a(x)\left(\left.\frac{\delta G^a[A_\alpha,\chi_\alpha]}{\delta\alpha^b}\right|_{\alpha=0}\right)c^b(x)\,. \tag{9}$$

[1] $\mathcal{D}A$ stands for the path integral over the field A.

4. General linear gauge-fixing

The most general linear constraint constructed from gauge fields' four-divergences and scalar fields is $\partial_\mu A^{a\mu} - K^a{}_i \chi_i$, which can be easily extended by including a nonlinear part

$$G^a = \partial_\mu A^{a\mu} - K^a{}_i\, \chi_i + G^a_{\mathrm{nl}}\,, \qquad (10)$$

with any real matrix $K^a{}_i$, since A^a_μ and χ_i are both real; G^a_{nl} should be real, too, and have the same mass dimension as the linear part. Any function of G^a will now constrain the gauge field, but for simplicity $\mathcal{L}_{\mathrm{gf}}$ should be a polynomial in the fields, which in turn tells us that it is quadratic in G[2]: $\mathcal{L}_{\mathrm{gf}} = -1/2\, G^a L^{ab} G^b$, where L^{ab} is any real square symmetric matrix and $-1/2$ is for later convenience. Note that G^a_{nl} will only give cubic and quartic terms and hence not influence the propagators.

So $\mathcal{L}_{\mathrm{gf}}$ together with Eq.(4) yields for the bilinear part:

$$\begin{aligned}
&\mathcal{L}_2 + \mathcal{L}_{\mathrm{gf2}} = \\
&-\frac{1}{2} A^a_\mu (-g^{\mu\nu}(\partial^2 + FF^T) + (1-L)\partial^\mu\partial^\nu)^{ab} A^b_\nu \\
&+\frac{1}{2}\chi_i(-\partial^2 - K^T L K - M)_{ij}\chi_j \\
&-\partial_\mu A^{a\mu}(F - LK)^a{}_i \chi_i\,,
\end{aligned} \qquad (11)$$

where we switched to matrix notation: $F^a{}_i = (F)^a{}_i$, $K^a{}_i = (K)^a{}_i$, $L^{ab} = (L)^{ab}$, $M_{ij} = (M)_{ij}$.

Requiring the resulting propagators to be diagonal in particle species, we get rid of the mixing term by requiring

$$LK = F \qquad \text{or} \qquad K = L^{-1}F\,, \qquad (12)$$

leaving only L as the independent matrix. Here, taking an inverse, we assume that L is non-singular, so we will have to take a limit if we want to consider some of the eigenvalues of L to be zero.

We can easily read off the propagators for the gauge, scalar, and ghosts fields from the quadratic parts of the Lagrangian, but in order to diagonalize them, we will have to put in the specifics of the Standard Model.

[2] First order of G should not appear in $\mathcal{L}_{\mathrm{gf}}$, because the total derivative term $\partial_\mu A^{a\mu}$ is irrelevant in the Lagrangian, and the first-order field terms χ_i would shift the minimum of the potential, so we have to redefine the fields again. Higher powers of G will have mass dimensions higher than 4, so that the gauge fixing would render the Lagrangian nonrenormalizable.

5. The bosons of the Standard Model

Ignoring the unbroken strong interaction, the Standard Model gauge group $G = SU(2)\times U(1)$ has structure constants (assuming t^4 is the generator of $U(1)$) $f^{abc} = \epsilon^{abc}$ for $a,b,c \in \{1,2,3\}$ and $f^{abc} = 0$ otherwise. The real scalar fields ϕ_i are taken to transform as the real components of the usual two-dimensional complex representation of $SU(2)$ with generators $t^a = 1/2\,\sigma^a$ for $a \in \{1,2,3\}$ and $t^4 = 1/2$, where σ^a are the Pauli matrices and t^4 is the generator of $U(1)$. We define the real components of the complex two-dimensional vector ϕ as

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} -\mathrm{i}\phi^1 & -\phi^2 \\ \phi^4 & +\mathrm{i}\phi^3 \end{pmatrix}\,, \qquad (13)$$

then the real representation matrices $T^a = -\mathrm{i}t^a$ in ϕ^i space become 4×4 matrices.

Following the previous section, we consider the symmetry breaking of this group, which happens because the scalar fields are subjected to some potential $V(\phi)$. A renormalizable potential that obeys the gauge symmetry and does not have a local minimum at $\phi = 0$, as required for symmetry breaking, is

$$V(\phi) = -\frac{\mu^2}{2}(\phi^i\phi^i) + \frac{\lambda}{4}(\phi^i\phi^i)^2\,, \qquad (14)$$

with some unknown parameters μ and λ. As a result, ϕ acquires a vacuum expectation value (VEV) at the minimum of V: $|\phi_{i0}| = |v_i| \equiv \sqrt{\mu^2/\lambda}$. Note that all directions of ϕ are equivalent under the gauge transformation, so any choice of direction for ϕ_{i0} gives the same results, therefore, we can arbitrarily choose it as $v_i = \delta_{i4} v$.

5.1. Vector bosons

The combination of generators $Q = T^3 + T^4$ leaves ϕ_{i0} invariant. Therefore, it is still a symmetry of the theory, which is identified with the electromagnetic charge. Using g and g' as coupling constants for the groups $SU(2)$ and $U(1)$, we get $F^a{}_i = g_a (T^a)_{ij} v_j = v/2\,(g\delta^a_i - g'\delta^a_4)(1-\delta_{i4})$. The gauge boson mass term FF^T in Eq. (11) can be diagonalized by

$$FF^T = U^\dagger M_A U = \mathrm{diag}(m_A^2)\,, \qquad (15)$$

with a block diagonal unitary matrix $U = \mathrm{diag}(U_{12}, U_{34})$, where

$$U_{12} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -\mathrm{i} \\ 1 & \mathrm{i} \end{pmatrix}, \quad U_{34} = \frac{1}{\sqrt{g'^2 + g^2}}\begin{pmatrix} g & -g' \\ g' & g \end{pmatrix}, \qquad (16)$$

giving the physical field combinations and their masses as

$$W^{\pm}_{\mu} \equiv A'^{1,2}_{\mu} = \frac{1}{\sqrt{2}}(A^1_\mu \mp \mathrm{i}A^2_\mu)\,,\ m_W = g\frac{v}{2}\,,$$

$$Z^0_\mu \equiv A'^3_\mu = \frac{1}{\sqrt{g^2+g'^2}}(gA^3_\mu - g'A^4_\mu)\,,$$

$$m_Z = \sqrt{g^2+g'^2}\frac{v}{2}\,,$$

$$A_\mu \equiv A'^4_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g'A^3_\mu + gA^4_\mu)\,,$$

$$m_A = 0\,. \qquad (17)$$

Replacing the two unknown coupling constants g and g' with the electric charge e and the Weinberg angle θ_w defined as

$$e = \frac{gg'}{\sqrt{g^2+g'^2}}\,,$$

$$c_W \equiv \cos\theta_w = \frac{g}{\sqrt{g^2+g'^2}}\,,$$

$$s_W \equiv \sin\theta_w = \frac{g'}{\sqrt{g^2+g'^2}}\,, \qquad (18)$$

we can express the covariant derivative in terms of the physical gauge fields [3]

$$D_\mu = \partial_\mu - \mathrm{i}\frac{g}{\sqrt{2}}(W^+_\mu \hat{t}^+ + W^-_\mu \hat{t}^-) - \mathrm{i}\frac{g}{\cos\theta_w}Z_\mu(\hat{t}^3 - s^2_W\hat{q}) - \mathrm{i}eA_\mu\hat{q}\,, \qquad (19)$$

with the new generators

$$\hat{t}^\pm \equiv (\hat{t}^1 \pm \mathrm{i}\hat{t}^2)\,, \qquad \hat{q} \equiv (\hat{t}^3 + \hat{t}^4)\,. \qquad (20)$$

This allows us to show that $W^\pm$ transform irreducibly under the charge operator $\hat{q}$. With the gauge fields transformed to the physical basis, the quadratic Lagrangian for them becomes

$$\mathcal{L}_A = -\frac{1}{2}(A'^*)^a_\mu\big[-g^{\mu\nu}(\partial^2 + m^2_A) + (1 - ULU^\dagger)\partial^\mu\partial^\nu\big]^{ab}A'^b_\nu\,, \qquad (21)$$

which should be diagonal in the fields. Therefore $L' \equiv ULU^\dagger$ has to be diagonal in this basis as well. Choosing the usual notation, $L' = \mathrm{diag}(1/\xi_+, 1/\xi_-, 1/\xi_Z, 1/\xi_A)$. This gives us the original matrix $L = U^\dagger L' U$. Since this matrix has to be symmetric, we conclude that $\xi_+ = \xi_- \equiv \xi_W$, which shows that we can have three independent linear gauge-fixing parameters.

[3] Note that $\hat{t}^i$ here are generators and depend on the representation.

Finally, the Lagrangian for gauge bosons in the physical basis becomes explicitly diagonal

$$\mathcal{L}_A = \sum_{X\in\{W^\pm,Z^0,A\}} -\frac{1}{2}X^*_\mu\big[-g^{\mu\nu}(\partial^2 + m^2_X) + (1-\xi_X)\partial^\mu\partial^\nu\big]X_\nu\,, \qquad (22)$$

and masses as in Eq. (17).

5.2. Goldstone bosons and Higgs

With $\phi_{i0} = v\delta_{i4}$ we get

$$M_{ij} = \big[-\mu^2\delta_{ij} + \lambda(\phi_k\phi_k)\delta_{ij} + 2\lambda\phi_i\phi_j\big]|_{\phi=\phi_{i0}} = (-\mu^2 + \lambda v^2)\delta_{ij} + 2\lambda v^2\delta_{i4}\delta_{j4} = 2\mu^2\delta_{i4}\delta_{j4}\,, \qquad (23)$$

which clearly satisfies

$$(T^a\phi_0)_i M_{ij} = F^a{}_i M_{ij} = 0 \qquad (24)$$

for all a, which is nothing but the Goldstone theorem. This tells us that the two terms in the quadratic scalar Lagrangian can be split into

$$\mathcal{L}_G = \frac{1}{2}\chi_m(-\partial^2 - F^T L^{-1} F)_{mn}\chi_n \qquad \text{and}$$

$$\mathcal{L}_H = \frac{1}{2}h(-\partial^2 - m^2_H)h\,, \qquad (25)$$

with $m_H = \sqrt{2}\mu$ and $\chi_4 \equiv h$, being the Higgs boson. The three remaining Goldstone bosons get the mass, defined by the gauge-fixing parameters. Using the same redefinition for Goldstone boson fields as for gauge bosons, $\chi^\pm = 1/\sqrt{2}\,(\chi_1 \mp i\chi_2)$, we see that these are indeed the charge eigenstates. So the Lagrangian in the diagonal form looks as follows:

$$\mathcal{L}_G = \sum_{X\in\{W^\pm,Z^0\}} \frac{1}{2}\chi_X(-\partial^2 - \xi_X m^2_X)\chi_X\,. \qquad (26)$$

5.3. Ghosts

Using the same unitary transformation U, Eq. (16), for the ghost fields

$$c'^a = U^{ab}c^b \quad \text{and} \quad \bar{c}'^a = \bar{c}^b(U^\dagger)^{ba}\,, \qquad (27)$$

we get the quadratic Lagrangian for the ghosts

$$\mathcal{L}_g = \sum_{X\in\{W^\pm,Z^0,A\}} \bar{c}_X(-\partial^2 - \xi_X m^2_X)c_X\,, \qquad (28)$$

and the masses similar as for the Goldstone bosons, $\sqrt{\xi_X} m_X$, where m_X is the mass of the corresponding gauge boson. However, there is also a massless ghost corresponding to the massless vector field A_μ.

5.4. Interactions

When writing now the interactions in the SM, we will only write the parts coming from the nonlinear gauge-fixing terms as all other parts can be found in the standard literature, like in [9].

After the diagonalization of the propagators it is convenient to transform the constraint functions to physical fields, too, which instead of Eq. (10) now give

$$G^+ = \partial_\mu W^{+\mu} - \xi_W m_W \chi^+ + G^+_{\mathrm{nl}}\,, \tag{29}$$

$$G^Z = \partial_\mu Z^\mu - \xi_Z m_Z \chi^0 + G^Z_{\mathrm{nl}}\,, \tag{30}$$

$$G^A = \partial_\mu A^\mu + G^A_{\mathrm{nl}}\,, \tag{31}$$

with the nonlinear gauge-fixing given by [5, 6]

$$G^+_{\mathrm{nl}} = -\mathrm{i}\tilde{\alpha} e (A_\mu W^{\mu+}) - \mathrm{i}\tilde{\beta}\frac{e c_W}{s_W}(Z_\mu W^{\mu+}) - \tilde{\delta}\frac{e\xi_W}{2s_W}(h\chi^+) + \mathrm{i}\tilde{\kappa}\frac{e\xi_W}{2s_W}(\chi_3\chi^+)\,, \tag{32}$$

$$G^Z_{\mathrm{nl}} = -\tilde{\varepsilon}\frac{e\xi_Z}{2s_W c_W}(h\chi_3)\,, \tag{33}$$

$$G^A_{\mathrm{nl}} = 0\,. \tag{34}$$

The direct multiplications in

$$\mathcal{L}_{\mathrm{gf}} = -\frac{1}{\xi_W}G^- G^+ - \frac{1}{2\xi_Z}(G^Z)^2 - \frac{1}{2\xi_A}(G^a)^2 \tag{35}$$

are straightforward and can be found in [10], only the interactions with the ghosts are more tricky. We begin with Eq. (27) and express the ghost Lagrangian directly in terms of the physical fields as

$$\mathcal{L}_{\mathrm{gh}} \equiv \bar{c}^a\left(\frac{\delta G^a_\alpha}{\delta\alpha^b}\right)(-g)c^b \tag{36}$$
$$= -\bar{c}'^a U^{ac}\left(\frac{\delta G^c_\alpha}{\delta\alpha^d}\right)(gU^\dagger)^{db}c'^b = -\bar{c}'^a\left(\frac{\delta G'^a_\alpha}{\delta\alpha'^b}\right)c'^b$$

with transformed G^a and α^a:

$$G'^a = U^{ab}G^b \quad \text{and} \quad \alpha'^a = (Ug^{-1})^{ab}\alpha^b\,, \tag{37}$$

which are just the physical constraints (Eq. (29)).

The variation in the physical gauge fields is

$$\delta A'^a_\mu = U^{ab}\delta A^b_\mu = U^{ad}\left(g^{-1}\partial_\mu\alpha^d + f^{dbc}A^b_\mu\alpha^c\right)$$
$$= \partial_\mu\alpha'^a + \left(U^{ad}f^{def}(U^\dagger)^{eb}(gU^\dagger)^{fc}\right)A'^b_\mu\alpha'^c\,. \tag{38}$$

After transforming the structure constants we get

$$\delta W^\pm_\mu = \partial_\mu\alpha^\pm \pm \mathrm{i}e\Big(W^\pm_\mu\alpha^A + \frac{c_W}{s_W}W^\pm_\mu\alpha^Z - A_\mu\alpha^\pm - \frac{c_W}{s_W}Z_\mu\alpha^\pm\Big)\,, \tag{39}$$

$$\delta Z_\mu = \partial_\mu\alpha^Z - \mathrm{i}e\frac{c_W}{s_W}\Big(W^+_\mu\alpha^- - W^-_\mu\alpha^+\Big)\,, \tag{40}$$

$$\delta A_\mu = \partial_\mu\alpha^A - \mathrm{i}e\Big(W^+_\mu\alpha^- - W^-_\mu\alpha^+\Big)\,. \tag{41}$$

The variations of the scalar fields in the physical basis is

$$\delta\chi'_i = U^\chi_{ik}\delta\chi_k = U^\chi_{ik}\big[-\alpha^a F^a{}_k + \alpha^a T^a_{kj}\chi_j\big]$$
$$= -\big[U^\chi_{ik}F^b{}_k(gU^\dagger)^{ba}\big]\alpha'^a - \big[U^\chi_{ik}T^b_{k\ell}(U^{\chi\dagger})_{\ell j}(gU^\dagger)^{ba}\big]\alpha'^a\chi'_j\,, \tag{42}$$

which results in

$$\delta\chi^\pm = -m_W\alpha^\pm \pm \frac{\mathrm{i}e}{2s_W}\Big(\frac{c_W^2 - s_W^2}{c_W}\chi^\pm\alpha^Z + 2s_W\chi^\pm\alpha^A - \chi_3\alpha^\pm \pm \mathrm{i}h\alpha^\pm\Big)\,, \tag{43}$$

$$\delta\chi_3 = -m_Z\alpha^Z - \frac{\mathrm{i}e}{2s_W}\Big(\chi^+\alpha^- - \chi^-\alpha^+ - \frac{\mathrm{i}}{c_W}h\alpha^Z\Big)\,, \tag{44}$$

$$\delta h = \frac{e}{2s_W}\Big(\chi^+\alpha^- + \chi^-\alpha^+ + \frac{1}{c_W}\chi_3\alpha^Z\Big)\,. \tag{45}$$

With these results the calculation of Eq. (9) is again straightforward, especially when noting that the operator $c'^a\delta/\delta\alpha'^a$ just replaces the α'^a in the variations with c'^a. It can be found in [10].

6. Nonlinear gauge-fixing in FeynArts

FeynArts / FormCalc [3] is a freely available open source package for *Mathematica* that performs automatic calculations of Feynman diagrams up to 2-loop level. The physical model for calculations in FeynArts is not built-in but provided by external input files, called *model files*, which contain the information about the theory. The existing definition of the Standard Model in FeynArts matches the one we have discussed in this work in the case of linear gauge-fixing, when we redefine the phases $(Z^\mu, c^Z, \bar{c}^Z) \to -(Z^\mu, c^Z, \bar{c}^Z)$, $\chi^\pm \to \mp\mathrm{i}\chi^\pm$, and rescale the antighosts $\bar{c}^X \to \xi_X^{-1/2}\bar{c}^X$.

For the nonlinear terms we also had to modify the *generic model file*, `Lorentz.gen`, that describes the kinematic structure of the model. We had to include the divergence in the vector-vector-vector- (VVV-) and scalar-scalar-vector- (SSV-) vertices and to include ghost-ghost-vector-vector- and scalar-scalar-ghost-ghost- vertices, as these would not appear in the linear gauge-fixing.

Since we extended the structure of existing vertices, we had to change all occuring VVV- and SSV-vertices in the *classes model file* `SM.mod` and add our additional terms, proportional to the parameters $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\delta}$, $\tilde{\kappa}$, and $\tilde{\varepsilon}$.

After the implementation we calculated a number of amplitudes to check if the result is gauge-independent. All tree-level amplitudes in $\gamma\gamma \to W^+W^-$, $ZZ \to W^-W^+$, $e^-e^+ \to W^-W^+$, and $e^-e^+ \to \mu^-\mu^+$ were gauge independent. At one-loop level we checked that the renormalization procedure used in FeynArts is not spoiled by the inclusion of our new gauge-dependent couplings: all two-point functions in the processes $\gamma\gamma \to W^+W^-$, $e^-e^+ \to W^-W^+$, and $e^-e^+ \to \mu^-\mu^+$ were finite.

Our program can be downloaded from `http://terra.ar.fi.lt/~garfield/SM/`.

7. Outlook

The implementation of the nonlinear gauge-fixing in FeynArts / FormCalc will help to check automatically the gauge invariance of extended renormalization schemes, like an extension of the complex mass scheme [11] or other schemes where it is not clear if the gauge invariance is preserved.

Since the Minimal Supersymmetric Standard Model has a very similar structure in the gauge-fixing sector, we will extend the nonlinear gauge-fixing to the MSSM, as well. There the issue of having unstable particles as incoming or outgoing particles is at the same time more accepted and more problematic due to the large possible widths and the mixing of the supersymmetric particles.

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References

[1] ALEPH Collaboration, Precision electroweak measurements on the Z resonance, Phys. Rep. **427**, 257 (2006), `arXiv:hep-ex/0509008`.

[2] J.A. Aguilar-Saavedra, A. Ali, B.C. Allanach et al., Supersymmetry parameter analysis: SPA convention and project, Eur. Phys. J. C **46**, 43 (2006), `arXiv:hep-ph/0511344`.

[3] T. Hahn and M. Perez-Victoria, Automatized one-loop calculations in four and D dimensions, Comput. Phys. Commun.**118**, 153 (1999), `arXiv:hep-ph/9807565`.

[4] A. Denner and T. Hahn, Radiative corrections to $W^+W^- \to W^+W^-$ in the electroweak standard model, Nucl. Phys. B **525**, 27 (1998), `arXiv:hep-ph/9711302`.

[5] G. Belanger, F. Boudjema, J. Fujimoto, T. Ishikawa, T. Kaneko, K. Kato, and Y. Shimizu, Automatic calculations in high energy physics and Grace at one-loop, Phys. Rep. **430**, 117 (2006), `arXiv:hep-ph/0308080`.

[6] F. Boudjema and E. Chopin, Double Higgs production at the linear colliders and the probing of the Higgs selfcoupling, Z. Phys. C **73**, 85 (1996). `arXiv:hep-ph/9507396`.

[7] M.E. Peskin and D.V. Schroeder, *An Introduction to Quantum Field Theory* (Addison–Wesley, Reading, MA, 1995).

[8] S. Weinberg, *The Quantum Theory of Fields. Vol. 2: Modern Applications*, (Cambridge University Press, Cambridge, UK, 1996).

[9] A. Denner, Techniques for calculation of electroweak radiative corrections at the one loop level and results for W physics at LEP-200, Fortschr. Phys. **41**, 307 (1993).

[10] J. Pašukonis, *Implementation of Non-Linear Gauge-Fixing in FeynArts Package*, Master Thesis (Vilnius University, Lithuania, 2007), `arXiv:hep-ph/0710.0159`.

[11] A. Denner and S. Dittmaier, The complex-mass scheme for perturbative calculations with unstable particles, Nucl. Phys. Proc. Suppl. **160**, 22 (2006), `arXiv:hep-ph/0605312`.

NETIESINĖS KALIBRUOTĖS FIKSAVIMAS FeynArts PAPROGRAMĖJE

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Santrauka

Pastaruoju metu tiksliems elementariųjų dalelių fizikos teoriniams skaičiavimams yra plačiai naudojami automatinio Feinmano diagramų generavimo ir skaičiavimo paketai, tokie kaip FeynArts arba GRACE. Su jais galima atlikti Standartinio modelio (SM) arba Minimalaus supersimetrinio standartinio modelio (MSSM) procesų amplitudžių skaičiavimus iki dviejų kilpų tikslumu, kas rankiniu būdu nėra įmanoma dėl labai didelio diagramų skaičiaus. Šio darbo tikslas buvo įgyvendinti tokių skaičiavimų FeynArts pakete rezultatų patikrinimą remiantis kalibruotės invariantiškumo principu, naudojant netiesines kalibruotes Standartiniame modelyje.

Apžvelgtas Standartinio modelio su netiesinėmis kalibruotėmis, pasiūlytomis Boudjema ir Chopin, teorinis išvedimas, skiriant ypatingą dėmesį kalibruotės fiksavimo procedūrai. Šios teorinės analizės pagrindinis rezultatas yra tas, kad be trijų tradicinių kalibruotės parametrų, aprašančių tiesinę kalibruotę, gaunami penki papildomi netiesinės kalibruotės parametrai, kurie įeina į naujus dėmenis, modifikuojančius Feinmano taisykles. Pateikiamas šio modelio įgyvendinimas FeynArts pakete ir juo atlikti skaičiavimai, kuriais parodoma, kad įvairių procesų amplitudės be kilpų korekcijų gaunamos nepriklausomos nuo kalibruotės. Parodyta, kad netiesinė kalibruotė nekeičia renormalizacijos procedūros.