

STRUCTURE OF THE GROUND STATE OF SIX NUCLEON NUCLEI

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The optimal superposition of the harmonic oscillator shell model configurations for the six nucleon nuclei ground state description is obtained. It has been shown that only one such superposition is possible. This result is in the complete accordance with the experimental observations stating that in the six nucleon system there is only one bound state.

Keywords: six nucleon nuclei, translationally invariant shell model, density matrix

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1. Introduction

The nuclear shell model, years ago introduced as significant simplification of the atomic nuclei description, operates with some leading configuration, i. e. with a defined set of one-nucleon states and a simple wave function dependent on one-nucleon degrees of freedom. This picture of independent nucleons moving in some central field provides an easy way of the wave function antisymmetrization but currently can be considered only as some phenomenological tool for the heavy nuclei description, where microscopic approach based on the Schrödinger equation with some potential of the nucleon–nucleon (NN) interaction cannot be applied.

For few-nucleon systems or even for the lightest atomic nuclei this model gives unacceptable results. Moreover, it cannot describe even the simplest atomic nucleus – deuteron – due to the problems with translational invariance of the wave function and the nonzero quadrupole moment of this nucleus, asking for the D-state, i. e. the excited configuration, presence in the wave function expansion.

So, for the microscopic description of atomic nuclei it is necessary to apply the variational methods like Green function Monte-Carlo [1] or to take into account the huge amount of the excited shell-model configurations. In the last case the best for the wave function expansion is the basis of three-dimensional harmonic oscillator (HO) functions. This basis allows construction of the translationally invariant wave function taking into account the finite number of the basic states,

produced by all allowed configurations at the given number of the HO quanta [2, 3]. Sometimes this particular way of the lightest nuclei description is called the shell-model approach (to be more precise, ‘no-core shell model’). However, even in this approach, operating with a huge amount of basic states, one again has to apply some phenomenological structures, defined as the effective potentials, due to extremely slow convergence of the expansions. The bare NN potentials require even more basic states for a more or less acceptable solution of the Schrödinger equation even for few-nucleon systems.

The goal of our paper is to find out how important for the ground and excited states’ description is the starting wave function, by taking into account the complete set of the basic states of the minimal HO configuration for the reasons mentioned above. Our approach is based on the reduced Hamiltonian expansion for the energy of an atomic nucleus employing the translationally invariant wave function. This simple expression for the eigenvalue of a realistic nuclear Hamiltonian gives information about the role of different two nucleon channels in ensuring the stability of atomic nuclei.

Let us start our investigation with the nuclei consisting of six nucleons, because they are the lightest systems having a few more or less stable low-lying states with the well-defined quantum numbers [4] (Fig. 1). In five nucleon system there are no bound states at all. The nuclei consisting of a few nucleons (deuteron, tritium, and alpha particle) have no low-lying excited states. Therefore, the six nucleon systems are the best

Table 1. Diagonal elements of density matrix of the six nucleon nuclei [7].

| $J^{\Pi T}$ | 1^+0 | | | 3^+0 | 0^+1 | | 2^+0 | 2^+1 | | 1^+1 |
|---------------|-------------------|------------------------------|-------------------|-------------------|-------------------|-------------------|------------------------------|------------------------------|-------------------|------------------------------|
| Configuration | $(\frac{3}{2})^2$ | $(\frac{3}{2})(\frac{1}{2})$ | $(\frac{1}{2})^2$ | $(\frac{3}{2})^2$ | $(\frac{3}{2})^2$ | $(\frac{1}{2})^2$ | $(\frac{3}{2})(\frac{1}{2})$ | $(\frac{3}{2})(\frac{1}{2})$ | $(\frac{3}{2})^2$ | $(\frac{3}{2})(\frac{1}{2})$ |
| RH state | | | | | | | | | | |
| 3S_1 | 17/54 | 44/135 | 44/135 | 1/3 | 3/10 | 3/10 | 1/3 | 3/10 | 3/10 | 3/10 |
| $^3S'_1$ | 1/81 | 8/405 | 1/810 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1S_0 | 3/10 | 3/10 | 3/10 | 3/10 | 29/90 | 14/45 | 3/10 | 29/90 | 14/45 | 3/10 |
| $^1S'_0$ | 0 | 0 | 0 | 0 | 1/45 | 1/90 | 0 | 0 | 0 | 0 |
| 1P_1 | 19/270 | 13/270 | 13/270 | 1/30 | 1/30 | 1/30 | 1/30 | 1/30 | 1/30 | 1/30 |
| 3P_0 | 0 | 1/20 | 1/10 | 0 | 0 | 1/10 | 1/20 | 1/20 | 0 | 13/180 |
| 3P_1 | 1/20 | 1/8 | 1/5 | 1/20 | 13/180 | 11/45 | 1/8 | 47/360 | 11/180 | 17/120 |
| 3P_2 | 1/4 | 1/8 | 0 | 1/4 | 1/4 | 0 | 1/8 | 17/120 | 17/60 | 11/72 |
| 1D_2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1/45 | 1/90 | 0 |
| 3D_1 | 1/405 | 1/162 | 2/81 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3D_2 | 0 | 0 | 0 | 0 | 0 | 0 | 1/30 | 0 | 0 | 0 |
| 3D_3 | 0 | 0 | 0 | 1/30 | 0 | 0 | 0 | 0 | 0 | 0 |

$^3S'_1$ and $^1S'_0$ are excited RH states.

Table 1. The configuration for this nucleus, recommended by the shell model, is $(00\frac{1}{2})^4(11\frac{3}{2})^2$. This configuration creates four states of the six nucleon system: $J^{\Pi T} = 1^+0, 3^+0, 0^+1, 2^+1$. All these states are observed experimentally (Fig. 1): the ground state $J^{\Pi T} = 1^+0$, the first excited state $J^{\Pi T} = 3^+0$, the second excited state $J^{\Pi T} = 0^+1$ serving also as the ground state of ^6He and of ^6Be nuclei, and the fourth excited state of ^6Li nucleus $J^{\Pi T} = 2^+1$. The explanation of the two missing-on-the-list low-lying states $J^{\Pi T} = 1^+0, 2^+0$ requires more complex configurations. The first among them is the configuration $(00\frac{1}{2})^4(11\frac{3}{2})(11\frac{1}{2})$, giving the mentioned states as well as the states $J^{\Pi T} = 1^+1, 2^+1$ not yet identified experimentally. The complete set of configurations present in the minimal HO shell model consists of those mentioned and one additional configuration, $(00\frac{1}{2})^4(11\frac{1}{2})^2$, that yield two states: $J^{\Pi T} = 1^+0, 0^+1$. Thus the minimal HO shell model approximation contains three configurations and gives ten states, exceeding the number of experimentally observed levels by four.

Due to peculiarities of the realistic nucleon–nucleon potential the minimal value of the RH operator is in the channel $j^{\pi t} = 1^+0$ (3S_1 – 3D_1), playing the major role in accumulating the binding energy of nucleus [6]. This minimal RH eigenvalue requires the significant 3D_1 state admixture (not less than 30%, different for various realistic potentials) to the 3S_1 state. The diagonal entries of the intrinsic density matrix presented in Table 1 show that some admixture of this state can be found only in three 1^+0 ^6Li states, therefore at first

glance it looks like the theory suggests that the ground and the first two excited states in the ^6Li nucleus should be the 1^+0 states. However, the experiment proves a different picture: one 1^+0 state appears as the ground state, the second is the sixth in the row, and the third one is not found at all. It is thus obvious that the theoretical spectrum in minimal shell model approximation of the ^6Li nucleus is noticeably different from the one observed experimentally.

Is it possible to find out what linear combination of the mentioned three configurations would serve as the best starting function for describing the ground state $J^{\Pi T} = 1^+0$ of this nucleus? The answer is given by the intrinsic density matrix diagonalization.

The intrinsic density matrix in the reduced Hamiltonian state 3D_1 is

$$\begin{pmatrix} \frac{1}{405} & -\frac{1}{81\sqrt{10}} & -\frac{\sqrt{2}}{81\sqrt{5}} \\ -\frac{1}{81\sqrt{10}} & \frac{1}{162} & \frac{1}{81} \\ -\frac{\sqrt{2}}{81\sqrt{5}} & \frac{1}{81} & \frac{2}{81} \end{pmatrix}. \quad (2)$$

It is the diagonal elements of this matrix that are given in Table 1. After the diagonalization of this density matrix the weight of the two nucleon state 3D_1 for one of the functions equals $1/30$, which is the maximum possible value, and the weights of this state in the diagonal density matrix of the remaining two states equals to zero (Table 2). The eigenvector corresponding to this

Table 2. Density matrix of the 1^+0 states of ${}^6\text{Li}$ after diagonalization.

| $J^\pi T$ | 1^+0 | | |
|-----------------------------|--------------|-----------------|-----------------|
| | ground state | excited state 1 | excited state 2 |
| Configuration superposition | | | |
| RH states | | | |
| ${}^3\text{S}_1$ | 1/3 | 23/70 | 32/105 |
| ${}^3\text{S}'_1$ | 0 | 1/35 | 1/210 |
| ${}^1\text{S}_0$ | 3/10 | 3/10 | 3/10 |
| ${}^1\text{S}'_0$ | 0 | 0 | 0 |
| ${}^1\text{P}_1$ | 1/3 | 3/70 | 19/210 |
| ${}^3\text{P}_0$ | 1/12 | 1/70 | 11/210 |
| ${}^3\text{P}_1$ | 7/40 | 1/14 | 9/70 |
| ${}^3\text{P}_2$ | 1/24 | 3/14 | 5/42 |
| ${}^1\text{D}_2$ | 0 | 0 | 0 |
| ${}^3\text{D}_1$ | 1/30 | 0 | 0 |
| ${}^3\text{D}_2$ | 0 | 0 | 0 |
| ${}^3\text{D}_3$ | 0 | 0 | 0 |

maximal weight of the ${}^3\text{D}_1$ state is

$$\begin{pmatrix} \frac{\sqrt{2}}{3\sqrt{3}} \\ -\frac{\sqrt{5}}{3\sqrt{3}} \\ \frac{2\sqrt{5}}{3\sqrt{3}} \end{pmatrix}. \quad (3)$$

Obviously, this vector is equal to a superposition of the three mentioned configurations, giving the best starting approximation for description of the ground state of the ${}^6\text{Li}$ nucleus. Using the same orthogonal matrix we diagonalize the density matrices in all the other channels and obtain new ones for ${}^6\text{Li}$, presented in Table 2.

The diagonal entries of the new matrices are the elements of the density matrix of translationally invariant shell model. After the diagonalization the second and the third 1^+0 states in the ${}^6\text{Li}$ nucleus become equivalent to all the other states of minimal approximation, i. e. they contain no ${}^3\text{D}_1$ admixture. Thus, after the diagonalization, only one of the independent configuration superpositions having the maximal ${}^3\text{D}_1$ state admixture corresponds to the best approximation for the ${}^6\text{Li}$ ground energy level description.

3. Discussion and conclusions

The obtained zero approximation for the ${}^6\text{Li}$ ground state wave function expansion in the translationally invariant shell model basis provides very interesting con-

clusions. Firstly, this result states that only one combination of the configurations in the minimal HO shell-model approximation has nonzero D-state admixture in the two-nucleon reduced Hamiltonian channel. The experimental situation corresponds to this result in the best possible way, because actually in the six-nucleon system there exists only one bound state – the ground state of the ${}^6\text{Li}$ nucleus. Some excited states are long-lived ones, but not completely bound. The second interesting result is that the obtained optimal superposition of configurations has the maximal weight equal to 20/27 of the configuration $(00\frac{1}{2})^4(11\frac{1}{2})^2$, while the configuration recommended by the HO shell-model $(00\frac{1}{2})^4(11\frac{3}{2})^2$ has ten times lower weight, equal to 2/27. The missing 5/27 is the weight of the last configuration $(00\frac{1}{2})^4(11\frac{3}{2})(11\frac{1}{2})$.

It is obvious that this component is a very important part of the complete wave function, but not the only one. The next terms in the expansion, corresponding to the superposition of the $E_{\min} + 2$ and the higher oscillator quanta configurations, are able to minimize the energy of the state to such an extent (as mentioned, in the RH channel ${}^3\text{S}_1$ – ${}^3\text{D}_1$ the ground state requires approximately 30% of the ${}^3\text{D}_1$ channel admixture, while the data in Table 2 show that the ${}^3\text{D}_1$ probability makes up only 10% in comparison with the weight of the ${}^3\text{S}_1$ state).

The problem with the negative quadrupole moment of the atomic nucleus ${}^6\text{Li}$ cannot be solved in this approximation, giving positive value, but it is this optimal zero approximation that makes it possible to obtain the negative value of the quadrupole moment taking into account the proper admixture of functions corresponding to the excited HO shell-model configurations.

Finally, this procedure does not ensure the minimization of the total energy and for a proper description of the six nucleon nuclei the admixtures of the excited shell model states in the wave functions expansion is necessary. These admixtures are unavoidable for the binding energy as well as for the negative value of the quadrupole moment in the description of the ground state of ${}^6\text{Li}$ nucleus.

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ŠEŠIŲ NUKLEONŲ BRANDUOLIŲ PAGRINDINĖS BŪSENOS SANDARA

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Santrauka

Teoriškai nagrinėjamos šešių nukleonų branduolių aprašymo minimaliame harmoninio osciliatoriaus sluoksnių modelio artitume galimybės. Pagrindinis šios šeimos branduolys yra ${}^6\text{Li}$ – lengviausias branduolys, turintis palyginus turtingą spektrą: vieną surištą ir dar penkias gana ilgai gyvuojančias energetines būsenas su gerai apibrėžtais kvantiniais skaičiais.

Tradicinis sluoksnių modelis, kai apsiribojama tik rekomenduojama pagrindine konfigūracija $(00\frac{1}{2})^4(11\frac{3}{2})^2$, nusako tik keturias tų stebimų būsenų. Jeigu imamos visos trys įmanomos šio modelio konfigūracijos, tokių būsenų atsiranda dešimt. Taigi, modelis susiduria su principiniais sunkumais aprašant jau patį pirmąjį branduolį, turintį šiek tiek turtingesnę spektrą.

Ši seniai žinoma problema darbe sprendžiama pasinaudojant sluoksnių modelio modifikacija, paremta redukuotinio hamiltoniano operatoriumi ir užtikrinančia banginių funkcijų transliacinę invariantiškumą. Ši modifikacija leidžia bet kokios branduolio būsenos energiją išreikšti redukuotinio hamiltoniano tikrinėmis vertėmis, padaugintomis iš atitinkamų tikimybių. Seniai žinoma,

kad pagrindinis vaidmuo užtikrinant atomų branduolių stabilumą tenka redukuotinio hamiltoniano kanalui ${}^3\text{S}_1-{}^3\text{D}_1$, – tam pačiam, kuris aprašo deuterono pagrindinę būseną. Optimali bet kurios branduolio būsenos energija užtikrinama minimizuojant šio kanalo redukuotinio hamiltoniano pagrindinės būsenos energiją. Šis procesas reikalauja tam tikros, lengvai gaunamos iš skaičiavimų ir kiek skirtingos skirtingiems realistiniams potencialams, ${}^3\text{D}_1$ būsenos priemaišos. Atomų branduolių būsenos, kuriose šios redukuotinio hamiltoniano priemaišos tikimybė lygi nuliui, turi nedaug galimybių būti surištosios.

Sukonstravę šešių nukleonų branduolių tankio matricą ir ją diagonalizavę gavome, kad tik viena būseną iš dešimties turi nelygią nuliui šią tikimybę. Taigi, ji ir yra pagrindinė pretendente aprašyti ${}^6\text{Li}$ branduolio pagrindinę būseną $J^{\pi}T = 1^{+}0$. Tai gerai atitinka eksperimente stebimus rezultatus. Nelaukta išvada, kuri seka iš mūsų tyrimų, yra ta, kad pagrindinį svorį gautoje optimalioje nulinio artinio banginėje funkcijoje turi ne sluoksnių modelio rekomenduojama, o kita konfigūracija, būtent, $(00\frac{1}{2})^4(11\frac{1}{2})^2$.