Time-delayed feedback control of periodic orbits with an odd-number of positive unstable Floquet multipliers

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Outline

- Introduction: time-delayed feedback control (TDFC)
- Some scenarios of the stabilization via TDFC
- Odd number limitation
- Comparison of the proportional feedback control and delayed feedback control
- Examples of the successful stabilizations: Lorenz and Chua systems
Time-delayed feedback control

**Autonomous system with a unstable periodic orbit (UPO):**

\[ \dot{x} = f(x) + K[x(t - \tau) - x(t)] \]

The delay time must be equal to the period of UPO.

\[ \tau = T \]

noninvasive control force


Example of stabilization of period-one UPO in Rossler system:

![Graphs showing stabilization process](image-url)
Some scenarios of the stabilization (I)

Movement of the Floquet multipliers in the complex plane:

\[ \dot{x} = f(x) + K[x(t - \tau) - x(t)] \]
Some scenarios of the stabilization (II)

Movement of the Floquet multipliers in the complex plane:

![Diagram showing the movement of Floquet multipliers in the complex plane.](image)
Some scenarios of the stabilization (III)

Movement of the Floquet multipliers in the complex plane:

$m$- number of real Floquet multipliers larger than unity in the free system

The orbit is unstable if:

$$(-1)^m \lim_{\tau \to T} \frac{\tau - T}{\tau - \Theta(K, \tau)} < 0$$

$$\beta$$

$$\beta = 1 + \sum_{ij} K_{ij} C_{ij}$$


Some scenarios of the stabilization (III)

Movement of the Floquet multipliers in the complex plane:

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$$ \beta = 1 + \sum_{ij} K_{ij} C_{ij} $$


Comparison of the proportional and the delay feedback controls in the Lorenz system

**Proportional feedback control (PFC):**
\[
\dot{x} = f(x) + g\tilde{K}[x_c(t) - x(t)]
\]

**Delay feedback control (DFC):**
\[
\dot{x} = f(x) + \kappa\tilde{K}[x(t - \tau) - x(t)]
\]

\[
\lambda_{DFC} = \lambda_{PFC}(g)
\]
\[
\kappa = \frac{g}{1 - \exp(-\lambda_{PFC}(g)T)}
\]

\[
\tilde{K} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

\[
\tilde{K} = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0.5 \\ 0 & 0 & 0 \end{bmatrix}
\]
Successful stabilization: Lorenz system

\[
\begin{align*}
\dot{x}_1 &= 10 [x_2 - x_1] \\
\dot{x}_2 &= x_1 [28 - x_3] - x_2 \\
\dot{x}_3 &= x_1 x_2 - \frac{8}{3} x_3
\end{align*}
\]

\[
\tilde{K} = \begin{bmatrix}
0 & 0 & 0 \\
-1 & 0 & 0.5 \\
0 & 0 & 0
\end{bmatrix}
\]

\[\kappa = 0.865\]

\[\Delta x_2 = x_2(t) - x_2(t - \tau)\]
Successful stabilization: Chua system

\[
\begin{align*}
\dot{x}_1 &= 9(x_2 - \phi(x)) \\
\dot{x}_2 &= x_1 - x_2 + x_3 \\
\dot{x}_2 &= -\frac{100}{7} x_2
\end{align*}
\]

**Nonlinear function:**

\[
\phi(x) = \frac{2}{7} x_1 - \frac{3}{14} \left( |x_1 + 1| - |x_1 - 1| \right)
\]

\[
\tilde{K} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0.3 \end{bmatrix}
\]

\[
\kappa = 1.2
\]
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Thank you for your attention